THE JOURNAL of the Acoustical Society of America

· CONTENTS ·

Program of the Eighth Meeting 171

CARL F. EYRING. Methods of Calculating the Average Coefficient of Sound Absorption 178

W. J. SETTE. A New Reverberation Time Formula 193

R. R. RIESZ. The Relationship Between Loudness and the Minimum Perceptible Increment of Intensity . . . 211

IRVING WOLFF AND FRANK MASSA. Use of Pressure Gradient Microphones for Acoustical Measurements 217

E. J. ABBOTT. Calibration of Condenser Microphones for Soundmeters 235

G. M. GIANNINI. Some Improvements on the Operation of the Bells of a Carillon 245

F. A. FIRESTONE. A New Analogy Between Mechanical and Electrical Systems 249

Book Reviews 268

Vol. IV JANUARY, 1933 No. 3

THE JOURNAL
of the Acoustical Society of America

Volume IV Number 3

JANUARY · 1933

PROGRAM OF THE EIGHTH MEETING

OF THE

ACOUSTICAL SOCIETY OF AMERICA

NOVEMBER 28–29, 1932

Lydia Mendelssohn Theatre, University of Michigan
Ann Arbor, Michigan

MONDAY, NOVEMBER 28th, 1932

10:00 A.M.

1. **Facts Developed in the Design and Construction of the Johns-Manville Acoustical Laboratory.** JOHN S. PARKINSON AND PAUL O. YOUNG, *Johns-Manville Corporation.*

A brief description of the Johns-Manville Acoustical Laboratory is given, with test data on the efficacy of various constructions employed and a description of certain modifications in test procedure.

2. **Measurement of Transmission Loss through Partition Walls.** E. H. BEDELL AND K. D. SWARTZEL, JR., *Bell Telephone Laboratories.*

This paper reviews the theory and describes the method used at Bell Telephone Laboratories of measuring the transmission loss through partition walls.

The partition to be tested is built into an opening between two adjacent but structurally isolated rooms. A loudspeaker acts as a source of sound in one room and a portion of the sound energy is transmitted into the second room through the test partition. The transmission loss is taken as

$$TL = L_1 - L_2 - 10 \log_{10} (a_2/A)$$

where L_1 and L_2 are the intensity levels in the source and test room respectively, expressed in db, a_2 is the absorption in the test room and A is the area of the partition.

The levels L_1 and L_2 are measured and plotted with a moving coil microphone and an automatic level recorder, and a beat frequency oscillator is used as a source of tone so that the frequency may be varied continuously. Measurements with a continuous variation in frequency enable resonances in the partition to be much more easily and quickly detected than is possible when measurements are made at discrete frequency intervals. Both pure and frequency modulated tones have been used for the measurements. Results of measurements on a few partitions are given.

3. Effect of Rotating Vanes in a Reverberation Room. V. L. CHRISLER AND CATHERINE E. MILLER.

Rotating vanes have been installed in the reverberation room at the Bureau of Standards with the result that the rate of decay of sound is logarithmic even when a highly absorbent sample is in the room.

Since the vanes have been installed the change in the apparent coefficient with the size and shape of the sample is much smaller than has been previously measured. It is believed that this is due to the fact that the sound is kept more uniformly distributed by the rotating vanes.

4. The Dependence of Measured Absorption Coefficients upon Position and Quantity of Material. S. K. WOLF AND W. J. SETTE, *Electrical Research Products, Inc.*

Absorption coefficients obtained from reverberation chamber measurements are dependent not only on the amount of material present but also on the distribution of the material among the chamber surfaces. Curves obtained from experimental data are given to demonstrate the influence of these factors.

5. Electrically Produced Tone from String Vibration. LLOYD LOAR, *The Vivi-Tone Company.*

Several musical instruments will be demonstrated in which the small vibrations of strings are amplified to large volume.

MONDAY, NOVEMBER 28th, 1932

2:00 P.M.

SYMPOSIUM ON SUPERSONICS

The following papers on Supersonics are in response to invitations from the Society.

6. The Methods and Results of Supersonic Interferometry. J. C. HUBBARD, *Johns Hopkins University.*

In the acoustics of audible sound the instruments of investigation, including sources and receivers, are generally small as compared with the wave-length. Diffraction effects, i.e., the spreading of the waves, becomes an all important factor. On the other hand, in the optics of visible light the instruments of investigation are in general large as compared with wave-lengths, and diffraction effects may in many cases be neglected.

With Langevin's discovery of the methods of producing beams of high-frequency compressional waves in liquids, with the comprehensive investigation of their properties by Boyle, and finally, with the development by Pierce of the acoustic interferometer, acoustics has been developed in the region of short wave-lengths, the instruments and methods becoming analogous in many respects to those of optics. The field of acoustic research has thus been enlarged by the acquisition of interferometric methods. Aside from numerous applications of immediate practical interest the new methods are of great scientific value since they furnish a new working tool in a frequency region of significance in molecular physics. Various methods are discussed and a summary given of important conclusions.

7. Some Chemical Aspects of the Dispersion and Absorption of Sound. W. T. RICHARDS, *Princeton University.*

Modern acoustical interferometry promises to provide a quantitative answer to several questions of importance for the collision theory of chemical reaction rates. From the dispersion and absorption of sound in nitrogen tetroxide it is, for example, possible to obtain accurate and plausible values for the rate constant, the activation energy, and the effective molecular collision diameter of the dissociation reaction. Any other rapid chemical reaction should yield similar information from acoustical measurements. Since, according to current hypothesis reaction is closely analogous to the excitation of the vibrational heat capacity, the dispersion and absorption of sound in nondissociating gases has also direct chemical bearing. Measurements on the dispersion of sound in carbon dioxide, carbon disulfide, sulfur dioxide, and ethylene at various temperatures, pressures, and frequencies are discussed from this viewpoint. The velocity of sound in mixtures of ethylene with argon, nitrogen and hydrogen is also briefly considered. It is found that while collisions with argon and nitrogen are without influence on the vibrational heat capacity of ethylene, those with hydrogen are more effective than those with ethylene itself. This result has already been foreshadowed in the study of chemical reaction rates.

8. The Supersonic Interferometer and Absorption Measurements. W. D. HERSHBERGER, *Signal Corps Laboratories, Fort Monmouth, New Jersey.*

The previously developed theory of the supersonic interferometer is extended with the particular view of utilizing it for absorption measurements in gases. It is shown that the mechanical impedance of the interferometer is

$$Z = Z_0 + Z(l)$$

where

$$Z_0 = j(m\omega - s/\omega) + r$$

$$Z(l) = \frac{Ac\rho}{\tanh 2\alpha l} + \frac{Ac\rho}{\sinh 2\alpha l} e^{i\theta}$$

$$\theta = -\arctan \frac{\sin 2Kl \sinh 2\alpha l}{\cos 2Kl \cosh 2\alpha l - 1}.$$

That is, if reactance is plotted against resistance we obtain an impedance spiral with center $(Ac\rho/\tanh 2\alpha l, 0)$ and radius $Ac\rho/\sinh 2\alpha l$. The action of the interferometer is studied in terms of the properties of the spiral.

Absorption measurements are effected by compensating for the changes in the equivalent series electrical resistance due to changes in 1 by adjusting a shunt resistance in the

circuit so the e.m.f. across the crystal is held constant, irrespective of 1. Absorption in the medium is calculated from the values of shunt resistance for the path lengths of interest. Measurements were taken at five frequencies in the range 298.6 k.c. to 2514 k.c. The quantity $2\alpha\lambda^2$ as measured by the interferometric method is not constant but shows a systematic change with frequency. The possible significance of that change is pointed out.

9. Velocity and Absorption Measurements at Supersonic Frequencies. W. H. PIELEMEIER, *Pennsylvania State College.*

Importance of Supersonics. This field is related in varying degrees to gas theory, thermodynamics, chemistry, physical chemistry, mechanics, heat, radio engineering, navigation and biology.

The author's work in this field began in 1919 and has been confined to a search for a suitable high-frequency source and to the measurement of velocity and absorption in air, in oxygen and in carbon dioxide. Quartz slabs, properly cut from the original crystal and sputtered on both faces with metal were found far superior to the other tested sources for measurements requiring constant frequency and intensity. With such crystals, there were obtained frequencies ranging from 300 to 2000 k.c.

The velocity in air was found to be approximately 331.6 m/sec. in this range. At the lower frequencies the absorption appeared to exceed that predicted by the earlier theory. No serious deviation from expected values were found in oxygen. With carbon dioxide the velocity was found to increase about 3 percent from 300 to 2000 k.c. The absorption greatly exceeds the values predicted by the earlier theory. Both velocity and absorption values are in good agreement with those obtained theoretically by Kneser.

10. Studies in Supersonics. C. D. REID, *Rochester, New York.*

The paper naturally divides itself into two parts. The first deals with experiments concerning the energy distribution in a supersonic beam in water and describes a useful visual method of determining such distribution. Diffraction and interference phenomena similar to those in light are discussed. In the second part the method used for the precision measurement of supersonic velocity in air is described and the results discussed.

MONDAY, NOVEMBER 28th, 1932

8:15 P.M.

SPECIAL LECTURE

in the

LYDIA MENDELSSOHN THEATRE

11. Anecdotal History of the Science of Sound, with Some Personal Reminiscences. Illustrated with rare books, portraits, views and instruments. DAYTON C. MILLER, *Case School of Applied Science.*

TUESDAY, NOVEMBER 29th, 1932

9:30 A.M.

12. The Resonant Characteristics of Soft-Walled Cavities. J. C. COTTON, *Ohio State University, Introduced by G. Oscar Russell.*

Resonance is undoubtedly of great importance in voice production, but little attention has been given to the resonant characteristics of such cavities as are found in the human vocal mechanism. A soft-walled cavity obviously must react quite differently to a complex tone than does the ordinary hard-walled resonator. X-ray photographs showing the human vocal cavities and motion pictures of the vocal cords indicate that more than simple resonant reenforcement must be responsible in creating voice differences. Without question, absorption plays an important part.

It is now possible to determine quantitatively the effect of soft-walled cavities on complex tones by means of an electro-acoustical set-up developed in the Phonetics Laboratories at Columbus. A beat-note oscillator, a sound source of the loudspeaker type, and a potentiometer arrangement serve to produce tones of constant intensity from 75 to 10,000 cycles. A condenser microphone, a resistance coupled amplifier, and a copper oxide voltmeter indicate the sound intensity.

The cavity under investigation is coupled to the sound source and the frequency slowly varied over any portion of the afore-mentioned frequency range. Deviations of the frequency-response curve from a straight line indicate directly the frequency characteristics of the cavity.

13. Vocal Chords Control Pitch Only and Not Vocal or Vowel Quality. Demonstration. JOHN BELLAMY TAYLOR, *Schenectady, New York.*

Vibrating columns of air associated with wind instruments are seldom simple or even single. In a typical instrument, especially of the "reed" or "brass" classification, besides the air column that is usually considered, there is a second column or cavity on the air supply side of the "check-valve" or member which by opening and closing, releases air intermittently from bellows or lungs. This check-valve may be recognized as reed, tongue, lips, membranes or vocal cords. This air-column or cavity back of the check-valve has some control over the time of vibration, as was demonstrated by the author at Cleveland meeting of the Society, December 1st, 1931.

Usually the vibrations of the supply-side air column are heard not at all or but faintly, because of restricted opportunity for sound emission. But special devices may be employed (the stethoscope is an example) to hear sound in supply side air column and reduce or effectively suppress sound from external air column (the only one ordinarily considered).

With a microphone in close contact with an individual's chest, an amplifier and loudspeaker will make audible the sounds picked up from internal air column, and with sufficient amplification, the normal sounds of speech and singing are faint in comparison with the other. This set-up demonstrates that while the individual has pitch control on vibrations of the internal air column, he does not and cannot change the character or quality of the supply-side tone by any of the vocal speech or musical efforts.

This result is taken to indicate that the vocal cords have but one mode of opening and closing (at a given pitch) and play no part in determining the character or quality of vowels and other "vocalized" sounds.

14. The Interference of Subjective Harmonics. ERNEST K. CHAPIN AND F. A. FIRESTONE, *University of Michigan.*

The purpose of the work reviewed in this paper was to devise a techique for producing interference of subjective harmonics under controlled conditions and to investigate the nature of some of the effects of such interference.

15. A New Portable Meter for Noise Measurement and Analysis. W. O. OSBON AND K. A. OPLINGER, *Westinghouse Electric and Manufacturing Company.*

This article describes a new noise meter whose development is based upon several years experience in the field of industrial noise measurements. Care has been taken to make the instrument simple to operate and rugged enough for portable use.

The characteristics of noise and hearing which have been used as a basis for designing the meter are discussed. A superheterodyne analyzer attachment is described which permits analysis of noises with a high degree of selectivity.

The noise meter may also be used for vibration measurements with a moving coil type pick-up which is described.

16. Sound Measurements on Experimental Airplane Mufflers. W. F. SNYDER, *Bureau of Standards.*

For the past few years the Sound Section of the Bureau of Standards has been cooperating with the Aeronautics Branch of the Department of Commerce in the development of practical methods of reducing the noise in airplanes. A part of this work has been the measurement in absolute intensities of the noise emitted by an airplane engine fitted with various types of experimental mufflers.

The propeller noise was eliminated by connecting the engine to a hydraulic dynamometer instead of the propeller. The dynamometer also served for the measurement of the horsepower. A muffler made of steel barrels placed in the ground allowed measurements to be made in the absence of exhaust noises.

The sound measuring equipment consisted of a calibrated condenser microphone, a sound meter and a set of band-pass filters. Measurements were made with no filter in the circuit and at the frequency bands 0–250, 250–500, 500–1500, 1500–3000 and 3000–etc. The microphone was placed at six different stations ranging in distances from 2 feet to 80 feet from the engine.

It is possible to reduce the exhaust noises by 15 or 20 decibels below that of the unsilenced engine without adding too much bulk or weight to the aircraft. The propeller noise, however, must be below that of the muffled engine for the muffler to become effective.

17. On Minimum Audible Sound Fields. L. J. SIVIAN AND S. D. WHITE, *Bell Telephone Laboratories, New York, N. Y.*

The minimum audible field (M.A.F.) has been determined from data taken on 14 ears over the frequency range from 100 to 15,000 c.p.s. The observer is placed in a sound field which is substantially that of a plane progressive wave, facing the source and listening monaurally. The M.A.F. is expressed as the intensity of the free field, measured prior to the insertion of the observer. Another type of threshold data refers to minimum audible pressures (M.A.P.) as measured at the observer's eardrum. The differences obviously to be expected between M.A.F. and M.A.P. values are due to

wave motion in the ear canal and to diffraction caused by the head. The M.A.F. data are discussed in relation to the M.A.P. determinations available from several sources. Some possible causes of difference between the two, which are due to experimental procedure and may add to the causes already mentioned, are pointed out. At 1000 c.p.s. the average M.A.F. observed is 1.9×10^{-16} watts per cm^2, corresponding to 2.8×10^{-4} bars or 71 db below 1 bar. Data are included to show the effect of observer's azimuth with respect to the wave front.

18. A Note on Supersonic Methods. E. G. RICHARDSON, *University of Durham, Newcastle on Tyne, England.*

In collaboration with two research students, I have in hand an experimental investigation of the dispersion and absorption of supersonic waves in gases, O_2, CO_2, SO_2, N_2O, etc., with particular reference to the effects of pressure and temperature of the gases on absorption. Besides the now familiar Pierce method (for the velocity) and torsion-vane method (for the absorption), methods employing hot-wires have been developed. It has been found, for instance, that the steady resistance of a heated filament placed near a piezoelectric quartz oscillator varies with the amplitude of the supersonic waves to which it is exposed. The instrument is calibrated by keeping the filament at a fixed point relative to the quartz, and applying different alternating differences of potential to the latter. After this, the wire may be used, while the quartz is driven at constant potential, to plot the amplitude in its neighborhood, and hence the absorption coefficient for the gas may be calculated. If the velocity is required the wave-length of the radiation is measured by means of *two* heated filaments, each connected to one of two primaries of a transformer, the induced current in the secondary, due to fluctuations in the two primary circuits, being measured by a thermoelement and sensitive galvanometer. If the two wires are spaced at one wave-length apart, the two trains of fluctuations are in phase. Having determined the wave-length by keeping one filament still, and moving the other relative to it, until maximum response of the galvanometer is attained, the frequency is measured on a wave meter, and so the average velocity of the radiation over this small space is obtained. At present, the latter method has not proved practicable above 200 kilocycles, owing to the limited response on the part of the hot wires, but with improved technique, it is hoped to use the method at higher frequencies. Further details will be published in due course.

METHODS OF CALCULATING THE AVERAGE COEFFICIENT OF SOUND ABSORPTION

By CARL F. EYRING
Brigham Young University

INTRODUCTION

In a paper, *A Modified Formula for Reverberation*, G. Millington[1] proposes a formula which should be given consideration because of its value in solving certain very special acoustical problems. It will be shown that he introduces the method of using the *geometric mean* rather than the *arithmetic mean* in calculating the average coefficient of absorption, but that he *does not change the unique form* of the reverberation time formula. His paper is valuable since it brings out once more the need to heed the words of Dr. Paul E. Sabine, "In giving the absorption coefficient of any material, one must specify carefully the test conditions." This may also be said of the paper, *A New Reverberation Formula* by W. J. Sette[2] which has appeared so far only as an abstract.*

When one places absorbing material in a reverberation chamber and then on the walls of an auditorium and assumes it to have in both places the same effective coefficient of absorption as worked out by some reverberation formula, he tacitly assumes that the absorbing material plays an *identical rôle* in both situations. The methods of averaging the coefficients of absorption suggested by Millington, Sette and Andree[3] can be tested out experimentally best when some technique of unquestionable validity is established to measure directly the exact percentage of sound energy absorbed at each reflection from the adsorbing material. It should be clearly kept in mind that the sound chamber method of measuring this percentage is not a direct method.

THEORETICAL CONSIDERATIONS

In the summary of the paper *Reverberation Time in "Dead" Rooms*,[4] the author made clear that any general reverberation time formula must permit various methods of averaging the coefficients of absorption. Quoting from this paper, "We have shown that a correct picture of

[1] G. Millington, J. Acous. Soc. Am. **IV,** 69–82 (1932).
[2] W. J. Sette, J. Acous. Soc. Am. **IV,** 8 (1932).
* Editorial Note. Mr. Sette's article appears in this issue of the Journal, p. 193
[3] C. A. Andree, J. Acous. Soc. Am. **III,** 535, (1932).
[4] C. F. Eyring, J. Acous. Soc. Am. **I,** 217–241 (1930).

sound absorption in an enclosed room leads to a general reverberation formula of the form

$$T = kV/-S\log_e(1-\alpha_a). \tag{1}$$

The manner of the absorption, which has been described in detail in this paper, is responsible for the logarithm factor and therefore the new form; the mean free path between reflections which is independent of the nature of the absorption controls and magnitude of k; and α_a is obtained by assigning proper weights to the various elements of surface. For a diffuse condition of sound energy, probably the condition most frequently encountered, $k=0.05$ and α_a is obtained from Eq. (3) (the weighted arithmetic mean); but for *each ordered condition of the waves a particular value of k must be used and α_a must be obtained by proper averaging.*" Quoting from page 220, "We shall, following Sabine's experimental results, assume that α_a defined by Eq. (3) (the weighted arithmetic mean) may be considered as the uniform absorption coefficient of the walls; yet we realize that for an ordered condition of the sound waves this procedure may need to be modified."

G. Millington concludes that a formula of the form

$$T = -0.05V/\sum S_1\log_e(1-\alpha_1) \tag{2}$$

should be used for certain very special cases when an ordered condition of the sound waves is maintained during decay. It will be shown that his analysis really leads to a special method of averaging the coefficients of absorption, but does not modify the form of the reverberation time equation.

To arrive at Millington's formula one of two idealized assumptions must be made. In one case the sound energy will need to be considered confined to a sort of "ray-packet" which travels from one kind of surface to another, the number of times a certain surface is struck being porportional to the ratio of its area to the total area of the enclosure. This ray-packet will need to remain intact—there is to be no spreading of the beam, and the sound intensity will be measured not at some particular position in the enclosure such as the location of a microphone, but in the ray-packet itself. If we represent the total surface by S, its m parts by $S_1, S_2, \cdots S_m$, and the coefficient of absorption respectively of these parts as, $\alpha_1, \alpha_2 \cdots \alpha_m$, then the chance that the beam will be reflected from S_k, for example, will be S_k/S. If E_0 represents the initial energy density in the ray-packet (not necessarily the average value in the enclosure) then the value after n reflections will be

$$E_n = E_0(R_1R_2R_3\cdots R_mR_1R_2\cdots \text{to } n \text{ factors}) \tag{3}$$

where $R_m = 1 - \alpha_m$, etc. At once it is clear that the value of E_n is a discontinuous function of the time, its value dropping at each reflection. This step-like decay curve which represents the true decay might be ironed out into a smooth curve which would represent the approximate nature of the decay, a sort of average way in which the sound intensity might be thought to fall off continuously. The smoothed curve would be written as

$$E_n = E_0 R_a^n \tag{4}$$

where n is a continuous function of the time and R_a is a proper average. Since Eqs. (3) and (4) must describe the same decay we have at once,

$$R_a = (R_1 R_2 \cdots R_m R_1 R_2 \cdots \text{to } n \text{ factors})^{1/n}. \tag{5}$$

This means that R_a is a *geometric mean*, and from this the average value α_a may be calculated.

If $m << n$ this value of R_a may, with limitations pointed out later, be used no matter what the sequence of the reflections. Hence Eq. (3) may be written as follows

$$E_n = E_0(R_1^{(S_1/S)n} R_2^{S(_2/S)n} \cdots R_m^{(S_m/S)n}). \tag{6}$$

This is essentially the same as the following equation given by Millington

$$E_t = E_0(1 - \alpha_1)^{S_1 vt/Sp}(1 - \alpha_2)^{S_2 vt/Sp} \cdots (1 - \alpha_m)^{S_m vt/Sp}. \tag{7}$$

In this equation he tacitly assumes that t is a continuous function. His equation does not give strictly the true decay equation, but rather a smoothed out curve which may be considered to fall off continuously. We are, therefore, justified in writing Eq. (7) in the form

$$E_n = E_0 R_a^n \tag{8}$$

in which

$$R_a = (R_1^{(S_1/S)n} R_2^{(S_2/S)(S_2/S)n} \cdots R_m^{(S_m/S)n})^{1/n}. \tag{9}$$

If $m \geqq n$ the average value of the coefficient of absorption which should be used in Eq. (8) to best smooth out the discontinuous nature of the decay would certainly depend upon the sequence of the reflections. Even with $m << n$ limitations still exist as pointed out later.

This means that Mr. Millington's equation leads simply to Eq. (1)[5] the difference being that α_a is obtained from a geometric average, but never from an *arithmetic average*. Even for the idealized situation de-

[5] Eyring, reference 4, Eq. (24), p. 234.

scribed there is, therefore, no reason to assume the need of the distinct form he offers in Eq. (2).

The other picture which may be used to develop Millington's equation is one in which the sound energy is not confined to a ray-packet but uniformly distributed in the room. One of the m types of absorbing surface is made to cover completely the surface of the enclosure for the period of time required for sound to travel the mean free path between reflections. Then another sample is introduced for the next period, and so on during the decay of sound such that each type of absorbing material shall have been introduced as many times as its size warrants under the assumption that during n reflections a certain material will be used q times where q is the ratio of the size of the absorbing surface in question to the total area of the enclosure. The intensity of sound is arrived at by the same mathematical procedure used above. The average value of the coefficient of absorption is found to be the *geometric mean* as before, the only difference between the two idealizations being that in the first case the sound energy density is measured in the ray-packet, while in the second case it is measured at any position in the room.

If desired the method of images[4] may be used in this second case. As an approximate procedure, the source of sound and the walls of the enclosure may be replaced by the source of sound and image sources located in evenly spaced spherical zones—surfaces of concentric spheres separated by the distance p, the mean free path between reflections. The emissive power of each spherical image source will be constant over-all, but because of the assumed shift of the nature of the total surface of the room with each reflection, this magnitude will change from zone to zone in a manner dependent upon the order in which the various types of surfaces are introduced. The energy flow will be the same from all directions, so we sum for all directions and obtain the energy density in the enclosure when equilibrium is established as follows,

$$S_\infty = E_0(1 + R_1 + R_1R_2 + R_1R_2R_3 + R_1R_2R_3R_4 + \cdots). \quad (10)$$

As the number of factors in the terms increases, a cyclic recurrence of the values of R will be noted. This cycle may be long or short depending on the number of types of material and the area covered by each. During each cycle the decay will depend upon the order of appearance of the types of material into the room, but at the close of the cycle a properly weighted geometric mean may replace each of the values of R to obtain the correct energy density. During the next cycle this average

value may not be used to give the detailed decay, but at the close of the cycle it will again give the correct value. This process is repeated during the whole decay. At best then the weighted geometric mean represents the true average only at certain points on the decay curve. It is not certain that Millington recognized this limitation in his equation. He did recognize the limitations of the idealized picture which he proposed.

C. A. Andree[3] in a very interesting paper has clearly brought out the fact that various methods of averaging coefficients of reflection must be resorted to under different arrangements of materials and different assumptions as to the nature of the sound waves. His method suggests a means of obtaining an average value to represent the decay during the cycles discussed above. Such a value might be used instead of the geometric mean which holds only at the close of each cycle. But when the rate of decay of the sound intensity level in such a room is measured on a reverberation meter, which if either of these average values will the data yield? The problem is discussed in a general way in another part of this paper.

In the development of the general reverberation formula, the author[4] made use of the method of images. Throughout this discussion an average coefficient of reflection was used to represent the coefficients of the materials actually in the room. It was tacitly assumed that a certain type of absorbing material covered the whole surface of the enclosure. It was recognized then that care should be taken in obtaining the proper average to represent the various types of material. Since that time Andree[3] has pointed out the exacting conditions which need to be imposed if the weighted arithmetic mean of the coefficients of absorption is to be used without error. He[6] concludes that behind the using of this average in reverberation formulae, "lies the assumption that sound which is reflected from material having a given reflection coefficient has a probability of encountering, on its *next reflection*, material of (any) like coefficient which is equal to the ratio of the area of that material to the total area present." The author has also come to this same conclusion by an analysis based on the method of images.

Suppose that m types of absorbing materials with areas S_1, S_2, S_3, etc., cover the surface of the enclosure. As before, the walls of the room may be replaced by a series of image sources discretely located on concentric spheres spaced at a distance p apart. Because of dispersion the

[6] Reference 3, p. 546.

images will be blurred and hence it seems legitimate to assume spherical image sources instead of discrete images located on concentric spheres. Even for high frequencies where dispersion is least, this assumption is legitimate to a first approximation provided the sound is emitted from a point source. This discussion does not take into account the resonant effect of very small rooms.

It should be pointed out now as previously, that the actual energy density at a point in an enclosure produced by a point source may be divided into three parts; (1) that which is produced by the source only and this varies inversely as the square of the distance from the source; (2) that which is produced by the first few reflection image sources, and the point to point variation of intensity may be calculated for each particular enclosure; (3) that which is produced by the remaining sources which shows no point to point variation. When the rate of decay of the sound intensity level is determined by a meter, it is usually the decay of the third portion which is measured. It is to be understood, then, that the idealized picture proposed is to be applied only to this third portion, the first few image zones being strictly fictitious.

The emissive power of each image source will not be uniform over all, but will be made up of patches with different emissive power. The variety of patches becomes very great as the image source gets farther and farther from the source. The energy flowing into the enclosure from one direction, in a small cone $\Delta\omega$, will in general have a different composition than that flowing from another direction. Taking this into account we write for the total energy density in the enclosure at equilibrium the following equation,

$$
\begin{aligned}
S_\infty = {} & (E_0\Delta\omega/4\pi)(1 + R_1 + R_1R_2 + R_1R_2R_3 + \cdots) \\
& + (E_0\Delta\omega/4\pi)(1 + R_1 + R_1R_3 + R_1R_3R_4 + \cdots) \\
& + \cdots\cdots\cdots\cdots\cdots\cdots \\
& + \cdots\cdots\cdots\cdots\cdots\cdots \\
& + (E_0\Delta\omega/4\pi)(1 + R_2 + R_2R_1 + R_2R_3R_4 + \cdots) \\
& + (E_0\Delta\omega/4\pi)(1 + R_2 + R_2R_3 + R_2R_3R_5 + \cdots) \\
& + \cdots\cdots\cdots\cdots\cdots\cdots \\
& + \cdots\cdots\cdots\cdots\cdots\cdots \\
& + (E_0\Delta\omega/4\pi)(1 + R_m + R_mR_2R_3 + R_mR_3R_4 + \cdots) \\
& + \cdots\cdots\cdots\cdots\cdots\cdots \\
& + \cdots\cdots\cdots\cdots\cdots\cdots
\end{aligned}
\tag{11}
$$

The problem of averaging now presents itself. The second term in each series is a coefficient of reflection, and this coefficient is a common factor in all the remaining terms of the given series. If the weighted arithmetic mean,

$$R_a = (S_1R_1 + S_2R_2 + S_3R_3 + \cdots S_mR_m)/S, \tag{12}$$

where S is the total surface of the enclosure, is used to replace the second terms of all the series, then the point source must be considered placed at the center of a spherical room. Under this idealized situation, a given coefficient, R_2 for example, will appear as a common factor in $4\pi S_2/\Delta\omega S$ series, and at once the proper average will be the weighted arithmetic mean. A still further idealization must be resorted to if the first factor in all the other terms of the series is to be replaced by the value R_a. From an average point of view, the emissive power of the first image source may be considered uniform over-all. If the energy from this image source be focussed to a region in the enclosure and reemitted equally in all directions, thus playing the identical rôle of the original source except for the reduction in magnitude by the factor R_a, then R_a may replace the first factors in all the terms of the series. At once, after a rearrangement of the series, Eq. (11) takes the form,

$$\left.\begin{aligned} S_\infty = {} & (E_0\Delta\omega/4\pi)\{1 + R_a(1 + R_1 + R_1R_2 + \cdots)\} \\ & + (E_0\Delta\omega/4\pi)\{1 + R_a(1 + R_1 + R_1R_3 + \cdots)\} \\ & + \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ & + (E_0\Delta\omega/4\pi)\{1 + R_a(1 + R_m + R_mR_1 + \cdots)\} \end{aligned}\right\}. \tag{13}$$

The factors multiplied by R_a have exactly the form of the original series of Eq. (11), and the assumption and argument used to obtain (13) from (11) may again be used. As a result, after a rearrangement of the series we get,

$$\left.\begin{aligned} S_\infty = {} & (E_0\Delta\omega/4\pi)\{1 + R_a[1 + R_a(-)]\} + R_1 + R_1R_2 + \cdots)]\} \\ & + \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ & + \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ & + \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ & + (E_0\Delta\omega/4\pi)\{1 + R_a[1 + R_a(1 + R_m + R_mR_4 + \cdots)]\} \end{aligned}\right\}. \tag{14}$$

The process may now be repeated as many times as needed until the average value R_a only appears in the series and all the $4\pi/\Delta\omega$ series become identical. Summing, we get

$$S_\infty = E_0(1 + R_a + R_a^2 + \cdots R_a^n + \cdots), \tag{15}$$

the equation used in the former paper in the development of the reverberation formula.

It is important to clearly recognize the physical requirements back of this equation. The simple picture used in the above development tacitly assumes a point source at the center of a spherical room and the focussing of the energy remaining after each reflection into the equivalent of a point source. It is further assumed that the new source emits energy *identically like the original source*, that is, equally in all directions. This means that some sort of mixing mechanism must be postulated because, due to the nonuniform absorption of the wall surfaces, the energy flowing in certain directions into the region of focus will need to be augmented, and that coming from other directions will need to be decreased. But if by the use of the mixing mechanism, the energy focussed from a given element of area be reemitted equally in all directions, and if this be true of the energy from all elements of surface, then certainly, except in magnitude, the new source and the original source will become *identical*. Thus the requirements back of (15) will have been met, and the value R_a may be used without error. At once I may conclude with Andree that behind the rigorous use of the arithmetic average in reverberation formulae "lies the assumption that sound which is reflected from material having a given reflection coefficient has a probability of encountering, on its *next reflection*, material of *like coefficient* which is equal to the ratio of the area of that material to the total area present."

By analogy one would expect, therefore, that if a diffuse state were originally achieved, a state which would permit the accurate use of R_a for the first reflection—then *the exact nature of such a state would need to be maintained or reestablished after each reflection* in order to rigorously develop Eq. (15).

The art and technique of developing a diffuse state and the maintaining of it is at once before the experimenter. The error involved in the assumption of a diffuse state when this state is only approximated is another problem worthy of investigation. Experimental data point to the conclusion that with a careful distribution of absorbing material and the use of diffusing panels and paddles, a strictly diffuse state may be at least approximately established.

We now come to the problem of ascertaining the physical requirements imposed if Eq. (11) is to be averaged by the use of the weighted geometric mean. If a cyclic recurrence of the values of R is found in the large terms of Eq. (11), then each of these terms will take the form of

Eq. (10), and the geometric mean may replace the various values of R under the limitations pointed out above. But if the large terms of Eq. (11) are to be added in order that an equation of the form of (15) may be obtained, then not only must there be a cyclic recurrence of the values of R in each large term, but, if the recurrence comes with the m'th term in one of the large terms, it must come at the m'th term in all the other large terms. Further, these m'th terms, on the rearrangement of the factors which in general will be necessary, must be identical. As discussed above, unless this cyclic recurrence comes a very large number of times during the process of decay, the geometric mean can not be used accurately even under the exacting conditions just pointed out. This is especially true if it is to be used as rigorously as the arithmetic mean is used under the limitations discussed above.

I am unable even to suggest how such a general strictly ordered condition of the sound waves required for the use of the weighted geometric mean of all the absorbing material in an enclosure could be produced by a practical mechanism. With high frequencies and very directional sources, the sound energy might be approximately controlled so as to remain in a few strictly ordered states. For example, the sound could be made to move back and forth between two parallel walls, or diagonally about the room, each wall being struck in proper sequence and the corners of the room being carefully avoided. However, if the room is to be filled with energy and if all the walls are to be used, three distinct ordered conditions, a back and forth motion between all opposite wall, would need to be established. Thus, as pointed out by Andree, a combination of a geometric and arithmetic mean would need to be used. If an attempt were made to fill the enclosure with energy by sending a beam of sound diagonally about the room, the ordered condition would be vitiated at the corners.

All this means that in general Eq. (11) may not be averaged by a simple method. In a rough general manner the large terms of this equation may be arranged into three groups: terms which very nearly fulfill the conditions for the strictly diffuse state; terms which very nearly fulfill the conditions for an ordered state, and this group may have a number of subclasses; terms which cannot be classified with either of the other groups. In averaging the terms of the first group the weighted arithmetic mean may be used with little error. In the second group each subclass will have its geometric mean and the weighted arithmetic mean of these will be taken. For the third group, representing the state which is neither diffuse nor ordered, I am suggesting the use of an empirically

weighted arithmetic mean. (See Eq. (17).) Of course some other average could be experimentally tested and empirically established. The experimental aspect of this problem is discussed in the next part of the paper.

EXPERIMENTAL CONSIDERATIONS

In certain experiments the author has found situations[7] in which it seemed logical to assume part of the sound energy in a diffuse state and part in an ordered state. For high-frequency tones emitted by an exponential horn such a situation gave decay curves of the sound intensity level with two regions of distinct average slope. The first average slope, which represents in the main the decay of the original approximately diffuse sound, after a drop of 18 db change rather quickly to the second average slope which represents, in part at least, the decay of the sound which is reflected back and forth between two hard surfaced parallel walls. But a "beam of sound" travelling back and forth between the parallel walls without loss except at the reflecting surfaces would have had a reverberation time of eight seconds. Yet, at 1000 cycles the longest reverberation time measured was three seconds. This means that considerable energy left the beam of sound and was absorbed by the highly absorbing ceiling and floor. This indicates, I think, that the beam of sound is essentially a slowly diverging cone, and roughly speaking, a certain portion of the energy is passing into an approximate diffuse state from the more or less ordered condition of the sound waves.

Another interesting aspect of these experiments is that the average effective coefficient of absorption as calculated by the usual formula changed from 0.28 to 0.08 during the process of decay. If the ear method had been used a constant average value of 0.11 would have been reported. Placing diffusing panels on the hard surface walls eliminated the second and slow rate of decay of the sound intensity level. Under this diffuse condition the measured average coefficient of absorption, 0.28, checked closely with that calculated using reverberation chamber data and the assumption that the *average value should be the weighted arithmetic mean* of the various coefficients. Had the ear method been used the changing rate of decay would not have been detected and some other type of averaging would probably have been resorted to in order to obtain as low an average value as 0.11.

In another situation[8] I found that an open window approximating the size of a wall surface and large as compared to the wave-length could

[7] C. F. Eyring, J. Soc. Motion Picture Eng. **XV,** 528–548 (1930).
[8] C. F. Eyring, J. Acous. Soc. Am. **III,** 188 (1931).

not be considered to have a coefficient of absorption of unity in the application of the reverberation time formula for a diffuse state. "Since the reverberation time equation

$$T = 0.05V/-S\log_e(1-\alpha_a) \tag{16}$$

is built up on the assumption that a diffuse state is established in the enclosure, a second factor, the lack of a diffuse state, which gets worse as the opening becomes larger can account for the calculated decrease of the coefficient as the opening area increases. The method of calculation fails to give a coefficient of absorption of unity to the open window because the mean free path between reflections in this case differs from that established in an enclosure of the same size in which a diffuse state is reached. It fails also because the lack of a diffuse state makes it impossible for all elements of surface to have equal chances of registering on the sound waves their absorbing ability. The mode of wave travel between the parallel walls has a greater chance to survive than a mode which quickly takes the wave out through an open window. This means that a unit of surface on one of the parallel walls has a greater chance to register its absorbing ability on the sound waves than a unit of surface of the open window. This indicates at once that if Eq. (16) is to be used to calculate the reverberation time for this type of room and if a coefficient of absorption of unity is assigned to the open window then equal weights cannot be assigned to all elements of surface in the calculation of the average coefficient of absorption."

The more or less ordered or semi-diffuse condition of the sound waves in the two situations just described is produced in the first case because the sound source is directional for high frequencies, in the second case because the small room is not a true enclosure, and in both cases because of a concentration of the absorbing material. If a large area of very *highly absorbing material* approximating the size of a wall surface, were placed in a reverberation chamber, the room could hardly be called a true enclosure, and a condition of the sound waves would be produced which would be neither strictly diffuse nor strictly ordered. A concentration of absorbing material of any kind on a single wall would vitiate the accuracy of results obtained using the weighted arithmetic mean of the coefficients of absorption. Andree[3] has clearly pointed out this fact and has rightly given warning that a diffuse state should not be assumed as existing among the sound waves without good evidence. It is the opinion of the author, however, that the average coefficient of absorption can be obtained accurately from a reverberation chamber with

certainty only if the various formulae proposed for averaging can be tested by a technique which permits *the direct measurement* of the percentage of energy reflected at each encounter with a given type of absorbing material.

From all considerations it is clear that in order to include all possible cases one might in a general way divide the sound energy of an enclosure roughly into three parts: first, the part which is strictly diffuse; second, the part which is not strictly diffuse nor strictly ordered; third, the part which is strictly ordered. Let A, B, and C represent *the fractional part* of the total energy existing in these three states respectively, then we may write,

$$1 - \alpha_a = R_a = \frac{A \sum kR}{\sum k} + \frac{B \sum qR}{\sum q} + \frac{C \sum d(R_1^s R_2^t R_3^p \cdots R_m^v)^{1/r}}{\sum d}. \quad (17)$$

The values of k will usually correspond to the various areas of absorbing material, but because of "edge effects" this strict correspondence may not hold. The values of q will depend upon the type of sound source, the room configuration and the concentration of absorbing materials. Only after a long series of experiments with all types of sound sources and enclosures and all possible locations of absorbing material can the technique of determining even approximate values of q be established. If one can be sure that a portion of the sound energy moves about the room as ray-packets, then the third term must be included. At once we have for this term,

$$r = s + t + p + \cdots v. \quad (18)$$

Some of these exponents may be zero if it is found that certain of the surfaces are not used; and the weight d of each ray packet and the weight of each reflecting surface for each ray packet must be determined empirically.

In order to explain the experimental facts reported above for the small auditorium it must be possible under certain situations for A, B and C to be variables, functions of the time. In the case of the small auditorium, energy was continuously leaving the roughly ordered state and going into at least a semi-diffused state. By introducing the diffusing panels the coefficients B and C were made to vanish. In the small room with the open window referred to above, the coefficients A and C were probably very small. Millington thinks that in a small room of volume 1933 cu. ft. and a total surface of 932 sq. ft. and absorbing material with an area of 27.8 sq. ft. he was able to set up a condition such

that no strictly diffuse or semi-diffuse sound existed, and therefore that A and B were zero. Of course he could have obtained the smaller desired coefficient of absorption by taking into account "edge effects," thus using only the first term of the equation, or he could have used the second term only, adjusting the values of q to give the desired coefficient. But does either of these procedures, or even the one he used, represent the reality of the situation? How is one to determine?

In the discussion so far it has been assumed that the values of R are accurately known. Usually these values are obtained in a reverberation chamber. But when a person places absorbing material in such an enclosure and then on the walls of an auditorium and assumes it to have in both places the same reflection or absorption coefficient as worked out by some reverberation formula, he tacitly assumes that the absorbing material plays an identical rôle in both situations. If a person can be sure that the sound in the chamber is perfectly diffuse and remains so during the decay, he may use the reverberation time Eq. (1) to calculate the absorption coefficient provided this formula uses the mean free path calculated for a diffuse state, and an average coefficient of absorption obtained from an arithmetic mean. If there are no edge effects then the calculated value of the coefficient is probably the true absorption coefficient of the material, yet there is still some little uncertainty whether the reverberation time equation will give the exact coefficient of absorption, that is, the exact percentage of sound energy absorbed at each reflection from the absorbing material. If a technique of unquestionable validity could be established for the measurement of this percentage of absorption under the situation found in the reverberation chamber, then this last uncertainty would be eliminated.

Any kind of reverberation formula used to calculate an absorption coefficient from a reverberation time, may be used under the identical circumstances to calculate the reverberation time from the absorption data. In a word, it is simply the process of working forward through a mathematical procedure and then retracing the exact steps to the original data. Therefore, if the identical situation found in a chamber is also found in an auditorium then any kind of formula could be used with success even though it described a fictitious physical state. It is easy to see, therefore, if one should use the *geometric mean* suggested by Millington in determining the coefficient of absorption for a diffuse state in a reverberation chamber, and then use the same method of averaging in calculating the reverberation time in a rather live auditorium in which the sound is also diffuse, that the final results would not differ greatly

from those obtained using the true average value for a diffuse state, the *arithmetic mean*. Again, suppose absorption coefficients are obtained in a reverberation chamber where a diffuse condition of the sound waves is maintained, and suppose that a diffuse state is also set up in a live auditorium, a reverberation formula which very nearly but not exactly describes the true physical state could give excellent results in spite of its inaccuracies because of the almost identical rôles played by the absorbing material in the two situations. The need for the new reverberation time Eq. (1) became apparent only when the situation in the reverberation chamber and that in the sound stage were far from being identical —one room was very live, the other was very dead. This brings out clearly that the checking of measurements made in a reverberation chamber with those made in an auditorium cannot test with precision the validity of a reverberation time equation. It would be best to check it experimentally against coefficients of absorption which have been measured by some direct process.

In practice most reverberation chamber and auditorium situations are not identical. For this reason a reverberation time formula which truly describes the physical reality is much to be desired. In other words, a true not a fictitious relationship between coefficients of absorption and reverberation time should be determined for as many different situations as possible. We have a formula which describes very well, I think, a diffuse state, and if precautions are taken in a reverberation chamber to obtain a diffuse state and if this formula is used, then the absorption coefficients measured will be approximately correct, that is, they will quite truly represent the percentage of the energy absorbed at each reflection from the surface in question.

If "the duration of audibility of the residual sound is nearly the same in all parts of an auditorium," if it is "nearly independent of the position of the source," and if "the efficiency of an absorbent in reducing the residual sound is, under ordinary circumstances, nearly independent of its position,"[9] the sound will be at least approximately diffuse and we shall have good acoustic conditions. Thus the regular reverberation time formula for a diffuse state may be used with little error. However, in certain cases, this approximate "ideal" condition may not be obtained. If so, one must use empirically some such equation as (16) to determine the average coefficient of absorption, and determine empirically[10] the

[9] Wallace C. Sabine, *Collected Papers*, pp. 18, 104.
[10] V. O. Knudsen, J. Acous. Soc. Am. **III**, 314 (1932).

value of k (a value which depends upon the mean free path between reflections) which should be used in Eq. (1).

Millington does not give data to prove that edge effects or that a semi-diffuse state could not explain his results. In other words, he does not show that he would not have been justified in weighting the surfaces in such a manner as to get an *arithmetic mean* of the smaller magnitude he desired. I am not willing to assume with him that in my experiments the unique state of the sound waves required by his *geometric mean* existed in fact. It may be that the sound was not in a perfect diffuse state, but I am convinced that it was approximately in that state. Millington does do this, he sets the minimum for my values, but the true value I am sure is much nearer the maximum which I give than the minimum which he calculates. My explanation of obtaining coefficients greater than unity, still holds. The difficulty of obtaining correct coefficients with high precision in a highly reverberent room is recognized by all who have made such measurements. Certainly I may claim observational error in measuring coefficients of absorption in a very dead reverberation chamber having a reverberation time of 0.34 second!

In conclusion I wish to emphasize that improvement in reverberation chamber technique should be in the direction of giving greater care to the establishment of a diffuse state among the sound waves, thus giving rise to the use of the weighted arithmetic mean of the coefficients of absorption and a value of the mean free path between reflections which is backed by well established theory. If it is found *impossible* under certain circumstances to establish this ideal state—this situation would be encountered more often in auditoriums than in sound chambers—then the methods of averaging suggested by Millington, Settee and Andree, when tested experimentally, will be of great value.

Acknowledgment is extended to Mr. Clarence A. Andree whose suggestions have stimulated the development of certain portions of this paper.

A NEW REVERBERATION TIME FORMULA

By W. J. Sette*
Electrical Research Products, Inc., New York City
(Received January 23, 1933)

Introduction

The earliest work relating decay of sound in an auditorium and the acoustic absorption of the surfaces was done by W. C. Sabine who developed the formula which has recently been shown to be applicable to only "live" rooms. More recently Fokker in Holland,[1] Schuster and Waetzmann in Germany[2] and Eyring in this country[3] derived an expression to hold for "dead" rooms also. The assumption of continuous absorption at the auditorium boundaries made in the Sabine formula was replaced by the conception of intermittent absorption, which is more in accord with actual conditions of decay.

Both of these formulae presuppose in their derivation uniform distribution of energy at each incidence, although Eyring observed that ordered states would necessitate assigning proper weights in computing the average surface absorption. The new formula is based on a similar assumption, but shifts the point of view to another kind of uniform distribution. Instead of each surface receiving a proportional share of the total energy in the room at each reflection, it is assumed that any ray of sound, after repeated reflection will have struck any one surface in proportion to the ratio of the area of that surface to the total room surface. This formulation of the process of decay leads to an alternative reverberation equation and some further extension of reverberation theory. The new equation is, of course, necessarily specialized and limited to those instances where the fundamental assumptions are fulfilled, as is brought out in the body of the paper, and under "Conclusion."

The following definitions and relations will be assumed in this work: V, room volume in cubic feet; S, total surface in square feet; S_x, any individual surface, k in all; a_x, acoustic absorption coefficient of S_x; R_x, $(1-a_x)$ = Reflection coefficient of S_x; T, time for 60 db decay in total sound energy; n, number of successive reflections in time T; p, Mean

* Presented at the May, 1932 meeting of the Acoustical Society. A recent paper by G. Millington, J. Acous. Soc. Am. **IV**, 69 (1932), presents a similar treatment of reverberation.

[1] A. D. Fokker, Physica **4**, 262 (1924).

[2] K. Schuster and E. Waetzmann, Ann. d. Physik [5] **1**, 671 (1929).

[3] C. F. Eyring, J. Acous. Soc. Am. **1**, 217 (1930).

free path between reflections; $p=4V/S$;[4] c, 1100 ft./sec.—velocity of sound.

DERIVATION

There will first be given a special brief derivation of the new formula, anticipating a longer and more general treatment.

Let the total sound energy in an enclosure at steady state conditions be E. When the sound source supplying the energy ceases "speaking," there are E units in the room distributed at random in space and direction of motion, travelling about at the rate of 1100 feet per second. In the course of a period of time referred to as the mean free time between reflections, all the rays (see Appendix I) constituting the total E strike the bounding wall surfaces and are reflected with a loss of energy. The energy lost is determined by the acoustic absorption coefficients, and the loss suffered by the individual rays will depend on the absorption of the particular surfaces encountered. After a second period of time a second set of reflections will have taken place. And so on, through the process of decay. Rays will strike first one wall, then another, and another, until finally, for our purposes, the total energy has reached one-millionth of its starting value.

Let us suppose that the energy originally contained in a particular beam is e_x. After incidence on a surface, say S_1, of reflection coefficient R_1, there will remain the fraction R_1 of e_x. If next this ray strikes another surface, S_2, of reflection coefficient R_2, there will remain the fraction of e_x, R_1 times R_2. After n reflections there is left the amount:

$$e_x R_1^{n_{x1}} R_2^{n_{x2}} \cdots R_k^{n_{xk}} \equiv e_{xn}. \tag{1}$$

It is assumed that there are k surfaces of reflectivity, R_1, R_2, $\cdots R_k$, and that the ray will strike each one in some sort of sequence. The exponents n_{x1}, etc., denote the number of times each surface is struck, the coefficients appearing in the product that number of times. When n is large enough, it becomes probable that each surface has been struck at least once. For sufficiently large values of n and under other conditions to be discussed later, we may assign values to these exponents. Suppose each surface enters into the process according to the ratio of its area to the total room surface, S. Thus, if the area corresponding to R_1 is S_1, then of n incidences, nS_1/S will be on S_1; the other surfaces accordingly. Then:

$$n = n_{x1} + n_{x2} + \cdots + n_{xk} = nS_1/S + nS_2/S + \cdots + nS_k/S.$$

[4] E. Buckingham, Bur. Standards, Sci. Paper No. 506, 1925.

This is for any ray e_x. We will further suppose that what is true for one ray, will, on the average, be true for another, and accordingly for the total energy in the room. After n reflections therefore, there will remain:

$$E(R_1^{nS_1/S} \cdot R_2^{nS_2/S} \cdots R_k^{nS_k/S}) = E(R_1^{S_1} \cdot R_2^{S_2} \cdots R_k^{S_k})^{n/S}. \quad (2)$$

From this point on we follow the procedure made familiar in the derivation of previous reverberation formulae. We will define time of reverberation, T, as that interval required for the sound energy in an enclosure to fall off to one-millionth of its steady state value. If n is the corresponding number of reflections, and p the mean free path between reflections, then: $np = cT$; but $p = 4V/S$, so that $n = cTS/4V$, and

$$E(R_1^{S_1} \cdot R_2^{S_2} \cdots R_k^{S_k})^{cT/4V} = E \cdot 10^{-6}.$$

By taking the natural logarithm of both sides, and eliminating the E's:

$$(cT/4V)(S_1 \ln R_1 + S_2 \ln R_2 + \cdots + S_k \ln R_k) = -13.8$$

where $1n \equiv \log_e$. Solving for T, it follows that:

$$T = 0.05V/-\sum_{x=1}^{k} S_x \ln R_x. \quad (3)$$

The summation in the denominator is extended from S_1 to S_k.

We may leave the formula in this form, in terms of reflection factors, which in this analysis have been more fundamental than the absorption coefficients, or we may express it in terms of the latter, which have been customarily used heretofore:

$$T = 0.05V/-\sum_{x=1}^{k} S_x \ln (1 - a_x). \quad (4)$$

The formula may also be written as:

$$T = 0.05V/S \ln R_g \quad (5)$$

where R_g is the geometric mean of the reflection factors. This will be shown in the following discussion.

DISCUSSION 1

For live rooms the expression derived by W. C. Sabine and others is readily obtainable. If the absorption coefficients of the surfaces of an auditorium are all supposed to be small, then the following equations hold in the limit as a_x approaches zero:

$$-\ln (1 - a_x) \doteq a_x$$

$$-\sum_{x=1}^{k} S_x \ln (1 - a_x) \doteq \sum_{x=1}^{k} a_x S_x = A$$

$$T = 0.05V/A. \quad (6)$$

To reduce the new formula to that derived by Eyring and others as a general one applicable to dead rooms, the assumption may be made that $a_1 \doteq a_2 \doteq \cdots \doteq a_x \doteq a_a$ whereby a_a is understood the arithmetic mean of the absorption coefficients over the surfaces. Then:

$$\ln (1 - a_1) \doteq \ln (1 - a_2) \doteq \cdots \doteq \ln (1 - a_a)$$

$$T = 0.05V/ - \sum_{x=1}^{k} S_x \ln (1 - a_a) = 0.05V/ - S \ln (1 - a_a). \quad (7)$$

The fundamental condition necessary for the new formula to reduce to the last one given is that:

$$\sum_{x=1}^{k} S_x \ln (1 - a_x) = \sum_{x=1}^{k} S_x \ln (1 - a_a).$$

We may examine this condition more fully by treating this equation. Divide both sides by S, take exponentials, and set $(1-a_x)=R_x$ and $(1-a_a)=R_a$. This leads to:

$$(R_1^{S_1} \cdot R_2^{S_2} \cdots R_k^{S_k})^{1/S} = R_a.$$

The expression on the left is the geometric mean, R_g in Eq. (5), of the reflection coefficients over the surfaces, that on the right the arithmetic mean. The new expression, therefore, will lead to the same result as the older one when the geometric mean of the coefficients is equal to the arithmetic mean. The new formula will be referred to as the geometric mean formula hereafter in this paper, and the older one as the arithmetic mean.

TABLE I. *Energy remaining.*

Reflection	R_1	R_2	Total	R_a	R_g
1	0.5	0.2	0.7	0.7	0.63
2	0.1	0.1	0.2	0.245	0.2
3	0.05	0.02	0.07	0.086	0.063
4	0.01	0.01	0.02	0.0302	0.02
5	0.005	0.002	0.007	0.0106	0.0063
6	0.001	0.001	0.002	0.0037	0.002

The difference between the workings of the two expressions can be illustrated by an example. Suppose two units of energy oscillating in a tube between two absorbing end surfaces of reflection factors $R_1=0.5$, $R_2=0.2$. Then $R_a=0.35$, $R_g=0.316$. Suppose one unit is first incident on R_1, then R_2, back to R_1, etc., and the other first incident on R_2, then R_1, etc. Table I shows the energy after several reflections, on the basis of direct computation, and computations by using the arithmetic and geometric means.

The arithmetic mean is greater than the geometric and hence shows a greater amount of energy left after any number of reflections. After the first reflection the geometric mean more closely corresponds to the actual values, as must be expected in a simple case like this when only simple products are involved. The greatest difference between the two occurs when the coefficients being averaged are quite different in magnitude. This is just the case in the instance of absorption measurements, which will be discussed later. Ordinarily the difference may be quite small.

Since reflection coefficients are always positive quantities, the arithmetic mean is greater than or at least equal to the geometric mean (Cauchy's theorem). This follows directly as an extension of the well-known relation: $(a+b)/2 \geqq (ab)^{1/2}$. The two values are equal only when $a=b$; the geometric mean equals the arithmetic mean only when all surfaces are equally absorbing. At other times $R_a > R_g$ and $-1n\ R_a < -1n\ R_g$. Accordingly times computed from the latter are the smaller. As we shall see later, other factors alter the situation somewhat.

DISCUSSION II

When one or more reflection coefficients are zero, the new formula breaks down. For $\ln R_x = \ln 0 = -\infty$. Then

$$T = 0.05V/-\sum_{x=1}^{k} S_x \ln R_x = 0.05V/\infty = 0;$$

that is, the reverberation time computes zero if only a single square foot of absorbing material with 100 percent absorption is involved. This is obviously correct.

This failure of the formula appears to hamper severely its usefulness as the case of an open window, i.e., 100 percent absorption, occurs commonly in practical work. However, by referring back to the fundamental new assumption concerning the distribution of incidences, it will be seen that it is untenable under this condition and hence that the formula should not be expected to hold. When a ray of sound strikes a surface that is completely absorbing, it is annihilated. If it is so removed from the scene, we cannot suppose that it will go on striking other surfaces in proportion to the ratio of their areas to the total room surface. We cannot think in terms of proportional distribution of incidences among the surfaces if the rays are extinguished upon striking one surface.

It would appear, therefore, that theoretically, the new formula cannot be applied to this special case. However, if we deal only with measured

values for absorption coefficients, this difficulty will be mitigated, as will be seen from Eq. (12) and Table III, in the next section. As there indicated, measurements on open windows lead to apparent absorptivities of less than unity, when they are computed according to the geometric mean formula.

APPLICATION OF FORMULA

At first sight, the new formula would be expected to lead to smaller computed reverberation times because the geometric mean of the co-

TABLE II. *Comparison of reverberation times.*

Frequency	120	240	500	1000	2000	3500
			a. Studio #C1			
			Volume, 30,260 cu. ft.			
Tm	2.35	1.60	1.14	0.60	0.46	0.48
Ta	2.95	1.83	1.02	0.64	0.60	0.67
Tg	2.88	1.71	0.92	0.46	0.48	0.55
			b. Studio #C2			
			Volume, 11,260 cu. ft.			
Tm	1.80	1.34	0.92	0.54	0.46	0.58
Ta	2.60	1.73	0.95	0.53	0.52	0.57
Tg	2.59	1.64	0.83	0.41	0.41	0.47
			c. Studio #C3			
			Volume, 17,500 cu. ft.			
Tm	2.22	1.57	1.04	0.54	0.46	0.47
Ta	2.91	1.85	0.99	0.58	0.57	0.63
Tg	2.84	1.77	0.90	0.45	0.45	0.51
			d. Studio #C4			
			Volume, 11,200 cu. ft.			
Tm	1.71	1.32	0.82	0.47	0.40	0.45
Ta	2.45	1.51	0.77	0.39	0.40	0.46
Tg	2.17	1.46	0.73	0.36	0.35	0.41
			e. Studio #S9			
			Volume, 6,520 cu. ft.			
Tm	0.80	0.70	0.69	0.62	0.62	0.60
Ta	1.29	1.07	0.79	0.69	0.65	0.68
Tg	1.12	0.92	0.65	0.54	0.50	0.53

Tm=measured time, *Ta*=time computed according to arithmetic mean, *Tg*=time computed according to geometric mean with Sabine coefficients.

The agreement between measurements and computations is not very good. Because the geometric mean times are based on Sabine coefficients, they are too low in 17 instances. The arithmetic mean times are too great in 23 instances. Calculations using the Sabine formula would lead to still higher times, and accordingly, none were made.

efficients is involved. On the basis of values for absorption coefficients presently accepted, it does, as will be seen from a glance at Table II. This table gives the results for reverberation times of five broadcast studios as computed by the arithmetic and geometric mean formulae and as measured by a chronograph reverberation meter. The times figured from the geometric mean are always smaller than the arithmetic mean times, and frequently check the measured values more closely.

The use in the new formula of coefficients based on the Sabine formula must be examined. To be correct, we should employ values derived by applying the new formula to obtain coefficients from reverberation chamber absorption measurements. New values for the absorptivity of materials will result. To show this, we proceed in the following manner.

Let the period of a reverberation chamber without a test material be:

$$T_1 = 0.05V/ - \sum_{x=1}^{k} S_x \ln R_x.$$

Or for our purposes:

$$\sum_{x=1}^{k} S_x \ln R_x = - 0.05V/T_1. \tag{8}$$

Let S_t and R_t be the constants for the test material and T_2 the resulting reverberation time. Then, if R_i is the reflection factor of the surface replaced by the material, (usually small):

$$- S_t \ln R_i + \sum_{x=1}^{k} S_x \ln R_x + S_t \ln R_t = - 0.05V/T_2. \tag{9}$$

Subtracting 8 from 9:

$$- S_t \ln R_i + S_t \ln R_t = - 0.05V/T_2 + 0.05V/T_1.$$

Then, since: $-\ln R_i \doteq a_i$ for small values:

$$- \ln R_t = (0.05V/S_t)(1/T_2 - 1/T_1) + a_i. \tag{10}$$

In the past the Sabine formula has been used as applying to live rooms. If an equation corresponding to (10) had been worked out on that basis, the result would be, for what we may call the Sabine coefficient, a_{ts}:

$$a_{ts} = (0.05V/S_t)(1/T_2 - 1/T_1) + a_i. \tag{11}$$

If we compare the last two equations, we see that $a_{ts} = -\ln R_t$. Or

$$a_{ts} = - \ln (1 - a_t). \tag{12}$$

It is seen that the two formulae lead to different values for absorption coefficients. In calculation of coefficients from reverberation time meas-

urements a quantity is obtained which on the basis of the Sabine formula is just the coefficient. On the basis of the new expression this same quantity is set equal to the negative of the natural logarithm of one minus the coefficient, the latter then following from logarithm tables. A comparison of values appears in Table III below. The Sabine coefficients are uniformly greater. This may be one explanation of the well-known fact that coefficients greater than unity are frequently computed from reverberation chamber data. This could be the case inasmuch as $-\ln(1-a_t)$ may be any positive number or even infinite. For small absorption, the coefficients are nearly equal.

TABLE III.

a_{ts}	0.05	0.1	0.2	0.4	0.6	0.8	1.0	1.2	1.4	∞
a_t (geometric mean value)	0.049	0.095	0.18	0.33	0.45	0.55	0.63	0.70	0.75	1.00

As a result of this relation, the new formula employing the new coefficients yields the same results as the Sabine formula, with the corresponding coefficients. Consider:

$$T = 0.05V/\sum_{x=1}^{k} a_x S_x; \quad T = 0.05V/\sum_{x=1}^{k} S_x \ln(1 - a_x).$$

The a_x in the former is identical with $-\ln(1-a_x)$ in the latter, and in computation the two, therefore, give identical values. Accordingly, the new formula does not lead to lower computed times as was originally anticipated, and, consequently, some other reason must be advanced to explain why measured times are so frequently less than the computed values. Such an explanation will be offered, but first we will consider computations with the arithmetic mean formula.

In Appendix II there is given the mathematical analysis leading to the value of the absorption coefficient based on the arithmetic mean formula as applied to reverberation chamber measurements. The result is:

$$a_{ta} = 0.05(V/S_t)(1/T_2 - 1/T_1)(1 - (a_2 + a_1)/2) + a_i. \qquad (13)$$

where a_{ta} = the new arithmetic mean coefficient, a_1 = average absorption empty, and a_2 = average absorption with material. Comparison of this expression with that resulting from the Sabine reverberation time formula shows that the new coefficients are somewhat less than the Sabine coefficients because of the factor $(1-(a_2+a_1)/2)$, which will be a few percent less than unity. If the coefficients so obtained are substituted

in the corresponding formula, only slightly higher times will result than from the use of the Sabine coefficients. There is apparent no simple algebraic relation between times computed solely on the basis of either the arithmetic mean and the original Sabine formula system, such as exists between the latter and the geometric mean system. The arithmetic mean reverberation time values will be smaller. If geometric mean coefficients are used in the arithmetic mean formula, higher computed times will result.

TRUE VALUES OF COEFFICIENTS

It is axiomatic that the absorptivity of a material is a fixed physical quantity, depending, of course, upon the method of mounting. The different formulae lead to different values due to the fact that in applying the formulae, we are effectively interpreting experimental results in terms of one or another assumption concerning the nature of the process of sound energy decay. The arithmetic mean formula assumes a uniform distribution of energy at each set of incidences on the surfaces; the geometric mean assumes that each ray of energy will strike any one surface a number of times proportional to its area. If the conditions in reverberation chamber absorption tests correspond to either one or the other of these assumptions, then the appropriate formula should be used, and true values for the absorption should result. Either formula could be used correctly, providing the actual conditions were made to fit the underlying theory. Offhand, we have no reason for supposing that either of the fundamental assumptions more closely corresponds to actuality. It is true that the arithmetic mean formula leads to coefficients greater than unity.[5] However, it is also true that for open windows, for frequencies at which diffraction effects are negligible, we do not measure $-\ln(1-a_g)$ to be infinite as we should for $a_g=1.00$. These two facts indicate that neither formula may be generally applied without further study.

Such considerations lead to an examination of the correctness of our methods of performing absorption measurements, which we will now make.

In Table IV are shown the results of measurements reported by J. S. Parkinson.[6] Equal areas of the test material were measured, collected in a single group, and then scattered over a greater space on the

[5] For example, see C. F. Eyring, reference 3, p. 238. Eyring ascribes the discrepancies to experimental errors, but if so, the errors are all in the same direction, an unlikely situation.

[6] J. S. Parkinson, J. Acous. Soc. Am. **2**, 112 (1930).

reverberation chamber floor. When the material was grouped in a compact formation, rather small coefficients were obtained; when it was spread out in the form of a loose pattern containing open spaces, the apparent coefficients became greater, in some cases in excess of unity. It does not appear possible to attribute the variation to diffraction effects inasmuch as the variation is most pronounced at high frequencies

TABLE IV. *Typical absorption coefficients resulting from various spacings.*
(All values for 1″ felt.)

Area of unit	Width of spacing	Frequency					
		128	256	512	1024	2048	4096
2′×2′	2′	0.07	0.30	0.78	0.96	0.78	0.48
2′×2′	4′	0.11	0.32	0.93	1.17	0.86	0.62
1′×8′	1′	0.10	0.30	0.73	0.82	0.76	0.71
1′×8′	2′	0.11	0.30	0.78	0.94	0.79	0.70
1′×8′	3′	0.11	0.32	0.85	1.15	0.84	0.66
2′×8′	1′	0.09	0.30	0.74	0.79	0.68	0.60
2′×8′	2′	0.09	0.31	0.76	0.84	0.71	0.65
2′×8′	4′	0.11	0.31	0.79	1.05	0.74	0.60
6′×8′	—	0.11	0.31	0.59	0.68	0.58	0.46

where the wave-lengths concerned are smaller compared to the sample dimensions. On the basis of a diffraction explanation, we should expect a decreasing variation with increased frequency.

This phenomenon is related to the well-known fact that small samples of an absorbing material are measured to possess higher absorptivities than larger samples. Both effects are due to the fact that we measure, not the actual, absolute coefficients of absorption, but merely the apparent absorptivity of the material as it affects the decay of sound energy in a particular reverberation chamber. Some time ago, C. F. Eyring[7] pointed out the existence of multislope decay times in rooms in which there was an ununiform distribution of absorbing agents. Sound energy travelling in directions necessitating repeated incidence on highly absorbing surfaces was quickly absorbed, and a sharp drop appeared in the decay curve. Other energy was confined between reflecting walls and lingered to give a slower rate of decay.[8] The absorbing material was not equally effective on all sound rays, but only on those

[7] C. F. Eyring, J. S. M. P. E. **15**, 528 (1930).

[8] A recent paper by V. L. Chrisler and C. E. Miller, likewise presented at the May, 1932 meeting of the Society, also discusses this phenomenon of double slope decay curves during absorption measurements.

which struck it. Hence its apparent absorption is less. If it had been distributed throughout the test room, a uniform and more rapid decay would have resulted with a corresponding increase in the computed coefficients of absorption. Parkinson's results may be explained in this manner. The reduced variation at low frequencies could be caused by the originally low absorptivities, and the increased likelihood of diffraction as an aid in redistributing sound energy.

In Fig. 1 are shown data collected in the West Coast Laboratory of Electrical Research Products, Inc. on 2 inch Rock Wool. The size of the

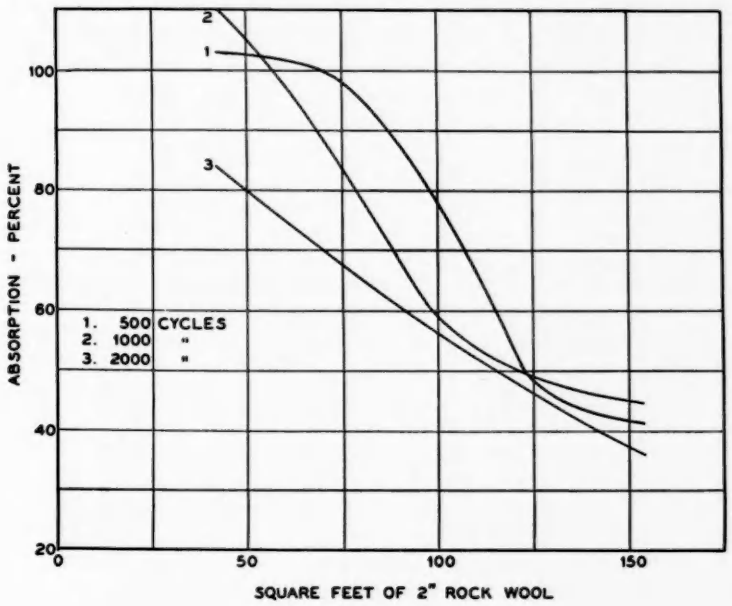


FIG. 1.

test sample was varied from 39 sq. ft. to 154 sq. ft., with a steady decrease in absorptivity. Computations were made on the basis of the Sabine formula. In two cases, the coefficient is greater than unity. Still smaller samples would have resulted in still higher coefficients, existing evidence indicates.

As previously indicated, the Sabine formula requires the assumption that sound energy is uniformly distributed at each incidence. This condition may be met, either by having so much of the absorbing material that it covers continuously most of the surface of the test chamber, or by having a lesser quantity distributed about the chamber in small pieces. Hence, the coefficients obtained with the smaller samples may be

taken as more nearly characteristic of the results to be expected from the use of the Sabine formula than those obtained with larger samples. Large isolated samples lead to nonlinear sound decay and exhibit what may be termed "local saturation" of absorbing power with a decrease in percentage effectiveness. The values for the small samples in Fig. 1 more nearly equal those obtained for 1 inch rock wool by C. F. Eyring[3] in tests where most of the chamber wall surface was covered by the material (approximating the first alternative condition mentioned above). This fact supports the conclusion that small samples lead to more characteristic results.

The Sabine coefficients at present in general use are based on tests of isolated samples of the order of 48 sq. ft., and according to the preceding analysis should be regarded as too low. In the studios for which reverberation times were given in Table II, the absorbent material was in general fairly well distributed, and therefore, more effective than under the usual test conditions. Hence, the reverberation times as computed on the basis of the conventional values for the Sabine coefficients would be too high. Furthermore, times computed according to the geometric mean formula, employing geometric mean coefficients arrived at by using Eq. (10) and the customary Sabine values, would also be too high.

The significance of this variation of measured absorptivity lies in indicating that the test conditions do not correspond to one or another simplifying assumption (uniform distribution of energy or of ray incidences) regarding the process of sound decay. Tests are now being conducted in which equal units of material are placed on three nonparallel walls of a room, and the results compared with those for single units on one and two walls. It is hoped that these tests will serve as a conclusive check of whether present coefficients are low because absorbing material placed on one wall only is not effective on all sound rays. It should be noted that where proper distribution of sound energy by rotating baffles or similar devices is achieved, such a condition should be minimized. Large test chambers would also be helpful, the condition being a function of relative size of sample and total wall surface.

Whether coefficients computed according to the geometric mean formula are closer to or farther from true values than those computed from the arithmetic mean formula remains to be decided. It is obvious that existing discrepancies render impossible a decisive choice in this respect. It is not even certain that the true values would lie between the limits given by the two computed values. For example, consider the coefficients computed for 154 square feet in Fig. 1. A conclusive check would

be had if it were feasible to measure coefficients with a material entirely covering the walls of a test chamber.

MORE GENERAL DERIVATION

As in the briefer treatment, let the total acoustic energy in a room at steady state conditions be E, and suppose that the source has just ceased supplying energy at the instant we begin our consideration. After the first reflection has occurred, the total energy remaining will depend upon the initial amount E, the distribution of incident energy among the individual surfaces, designated symbolically here by D, the individual reflection factors, and the areas of the surfaces possessing the separate reflection factors. In order to exactly determine the amount of energy left, each of these items must be known and computations made accordingly. For the present, we will be content with a symbolic formulation of the remainder. If we let U_1 represent this remainder after the first group of reflections, then we may express this inter-relationship algebraically in the following form:

$$U_1 = F^{(1)}(E, D, R_1, R_2, \cdots, R_k, S_1, S_2, \cdots, S_k).$$

where the superscript (1) denotes a function of first order complexity.

In the same way the decay of sound may be followed through the successive reflections. The second group of incidences will see rays of sound just reflected from S_1, for example, incident on S_2. The amount then reflected will be determined by R_2, the percentage of the original remaining after two reflections being proportional to $R_1 \cdot R_2$. For any one ray, the amount of energy left will be given by the product of the reflection factors of the surfaces struck, $R_x \cdot R_y$. The total energy remaining in the room after the second mean free period will thus depend on such products, the initial energy, E, the partition of energy and rays among the surfaces, and the individual areas. Symbolically we write:

$$U_2 = F^{(2)}(E, D, R_1, S_1 \cdots) \cdot$$

The superscript (2), denotes that second order products (R_1R_2, R_1R_3, etc.) are involved, and that the other variables figure in a second order relationship.

As the decay proceeds, our picture of the situation must become more complicated. After n successive reflections suffered by a particular ray of sound, its energy will have diminished to an amount proportional to the product of the reflection factors of the surfaces encountered in the process. If any surface is struck several times, its coefficient enters into

the product an equal number of times. The sum of the exponents of the coefficients in the product will equal the number of incidences. The total energy in the room after n reflections will be expressed by:

$$U_n = F^{(n)}(E, D, R_1, S_1, \cdots).$$

This equation represents a very general formulation of reverberation. It embodies general statements concerning energy remaining after n reflections. For the purpose of computation there is little that can be done with an expression of this form. Exact computation of sound decay would require a knowledge of the shape of a room, of the original space and direction distribution of energy, of the position of absorbing materials and of the manner in which reflected energy is directed, and would involve tedious, detailed calculations. However, easily usable and practically valuable expressions may be obtained provided reasonable assumptions are made as to average conditions. First we will consider further the nature of $F^{(n)}$. Its form is indicated by the following expansion:

$$F^{(n)}(E, D, R_1, \cdots) = (e_n)_1 R_1^{n_{11}} \cdot R_2^{n_{12}} \cdots R_k^{n_{1k}} + \cdots$$
$$+ (e_n)_x R_1^{n_{x1}} \cdots R_k^{n_{xk}} + \cdots + (e_n)_j R_1^{n_{j1}} \cdots R_k^{n_{jk}} \cdots$$

Here any $(e_n)_x$ is that amount of the original energy E which has undergone reflections as implied in the associated product, the sum of the small e's being equal to large E. The equation merely states that the different rays of sound will have j different histories of decay, and that the surfaces may enter into the process in different manners. As yet separate areas and their coefficients are not explicitly shown, these influencing the e's and the exponents of the R's.

The values of the n's must be ascertained. It will be first assumed that:

$$n_{11} + n_{12} + \cdots + n_{1k} = n_{21} + n_{22} + \cdots + n_{2k} = n_{x1} + \cdots + n_{xk} = n.$$

In words, each ray of sound is supposed to undergo the same total number of reflections. It is now time for the assumption concerning the partition of incidences, mentioned in the introduction: each surface will be supposed to be encountered by any sound ray in proportion to the ratio of its area to the total enclosed surface. For a total of n reflections, a surface S_x will be struck nS_x/S times. The distribution of incidences among the surfaces will be given by the following equations:

$$nS_1/S + nS_2/S + \cdots + nS_k/S = n$$
$$nS_1/S = n_{11} = n_{21} = n_{j1}$$

. . . . . .

. . . . . .

$$nS_k/S = n_{1k} = \cdots = n_{jk}.$$

It is necessary to record that for these assumptions to be tenable n must be at least equal to k, the number of different surfaces. There must occur enough reflections for each surface to participate at least once. A more rigorous condition is that $n > > S/S_x$. For random incidence the ratio S_x/S expresses the probability of any ray striking S_x, and gives the percentage of reflections in a sequence S of them which should involve S_x. Inversely, S/S_x gives the number of reflections that may occur before, on the basis of strict probability, it is certain that the surface S_x has been struck.

With this distribution of reflections, the remainder energy function takes the form:

$$\begin{aligned} U_n &= (e_{n1} + e_{n2} + \cdots + e_{nj}) R_1^{nS_1/S} \cdot R_2^{nS_2/S} \cdots R_k^{nS_k/S} \\ &= E(R_1^{S_1} \cdot R_2^{S_2} \cdots R_k^{S_k})^{n/S}. \end{aligned}$$

From this point on, we follow the procedure used in the first derivation.

Conclusion

The geometric mean reverberation time formula derived here is not offered as a general formula, but as a specific one to apply in those cases in which the fundamental assumption concerning the distribution of incidences of sound rays is fulfilled. That it does apply in some instances is evidenced by the fact that when used to compute absorption coefficients it never leads to values exceeding 100 percent, as may happen when other formulae are used. That neither it nor any other simple reverberation formula applies at other times is proved by sound decay curves which are nonlinear, as obtained by C. F. Eyring and others. For these latter cases, a formula is needed in which the rate of absorption is made a function of the time or the number of reflections elapsing. In the present work, however, a simplifying assumption was made, and the logical consequences of the assumption derived. The result is a special formula which on occasion can serve to obtain useful approximations.

Appendix I. Definition of Ray of Sound

In the derivation and subsequent discussion the term "ray" of sound has been used. By a ray is to be understood the energy in an elementary

volume, to be defined, which is travelling in a specified direction, and the subsequent space-time path described by this energy.

Let e_0 be the energy density at steady state, supposed uniform, with energy travelling in all directions. About a point P as a center imagine a small element of volume ΔV, which shall be made to approach zero. The energy contained so is $e_0 \cdot \Delta V$.

As the volume is made smaller, the energy leaving it will appear to emanate as though from a point source, for the volume approaches a point. The energy will be travelling in all directions spherically from the point. Now we select of $e_0 \cdot \Delta V$ that portion which is travelling in those directions included in an elementary solid angle Δw, with its apex at the point P. This portion is $\Delta e = e_0 \cdot \Delta V \, \Delta w/4\pi$.

This is what is to be understood by a ray of sound energy. The main consideration is that each such ray will travel, expand, and be reflected from the surfaces as an entity.

To make the above analysis more rigorous, it is necessary to pass to the limit and use differentials. Then: $de = e_0 \, dV \cdot dw/4\pi$.

In the derivation it is assumed that each elementary unit or ray behaves in the same manner, that each will strike the surfaces in proportion to their areas. After n reflections, there will remain of each ray, according to our theory, the fraction $R_g{}^n$, where R_g is the geometric mean of the reflection coefficients. If by U_n is understood the total remaining energy after n reflections, as in the more detailed second derivation, then:

$$
\begin{aligned}
d(U_n) &= e_0 \cdot dV \cdot dw/4\pi \cdot R_g{}^n \\
U_n &= \iiint_V \int_w e_0 dV (dw/4\pi) R_g{}^n \\
&= e_0 R_g{}^n \iiint_V \int_w dV (dw/4\pi) \\
&= e_0 R_g{}^n \iiint_V dV \\
&= e_0 R_g{}^n V \\
&= R_g{}^n E.
\end{aligned}
$$

From this point on the analysis is as in the paper. Because we fix our attention on a ray of sound energy before decay sets in, specular reflec-

tion of energy is assumed in the treatment, although it does not in general appear necessary.

APPENDIX II. APPLICATION OF AIRTHMETIC MEAN FORMULA TO ABSORPTION MEASUREMENTS

$$T_1 = 0.05(V/-S)1/\ln(1-a_1),$$
$$T_2 = 0.05(V/-S)1/\ln(1-a_2).$$

where by T_1 and T_2 are to be understood the decay times in a reverberation chamber, the former in its normal condition, the latter with test sound absorbing material. Rewriting these equations:

$$\ln(1-a_1) = -0.05(V/S)1/T_1$$
$$\ln(1-a_2) = -0.05(V/S)1/T_2.$$

Subtracting:

$$\ln(1-a_1) - \ln(1-a_2) = -0.05(V/S)(1/T_1 - 1/T_2).$$

Or, if we use differentials, as we may for small differences:

$$d(\ln(1-a)) = -0.05(V/S)d(1/T);$$

but:

$$d(\ln(1-a)) = -da/(1-a);$$

therefore:

$$-da/(1-a) = -0.05(V/S)d(1/T).$$

Then, using the customary calculus approximation method:

$$-(a_1-a_2)/[1-(a_1+a_2)/2] = -0.05(V/S)(1/T_1 - 1/T_2).$$

This is true providing a_2 and a_1 are not too greatly different. The validity of the approximation can be substantiated by direct numerical check of the equation:

$$\ln(1-a_1) - \ln(1-a_2) = -(a_1-a_2)/[1-(a_1+a_2)/2].$$

Proceeding with the main problem:

$$(a_2-a_1) = -0.05(V/S)(1/T_1 - 1/T_2)(1-(a_2+a_1)/2)$$

By definition:

$$a_2 = (a_1S - a_iS_t + a_tS_t)/S = a_1 + (S_t/S)(a_t - a_i);$$

S_t = area of test material;
a_i = absorption coefficient of area S_t displaced by test material;
a_t = absorption coefficient of test material.

Accordingly:

$$a_2 - a_1 = (S_t/S)(a_t - a_i)$$

$$(S_t/S)(a_t - a_i) = -0.05(V/S)(1/T_1 - 1/T_2)(1 - (a_2 + a_1)/2)$$

$$a_t = 0.05(V/S_t)(1/T_2 - 1/T_1)(1 - (a_2 + a_1)/2 + a_i)$$

which is the form given in the main body of the paper.

THE RELATIONSHIP BETWEEN LOUDNESS AND THE MINIMUM PERCEPTIBLE INCREMENT OF INTENSITY

By R. R. Riesz
Bell Telephone Laboratories

It has often been suggested that some relationship exists between the loudness of pure tones and the minimum perceptible increment of intensity but no such relationship has ever been established. For instance, the hypothesis that tones of different frequencies and whose intensities are the same number of minimum perceptible steps of intensity above the auditory threshold sound equally loud, does not agree with experimental observations. However, in this paper it is shown that with certain restrictions loudness and the minimum perceptible increment of intensity are related. Also, a discussion is given of the significance of recent measurements on the reduction of apparent loudness in the light of data available on the minimum perceptible increment of intensity.

If, for a pure tone of any frequency and of intensity E, the minimum perceptible increment of intensity be ΔE, then the ratio $\Delta E/E$ is known as the differential sensitivity corresponding to the intensity E.[1] The number of distinguishable steps of intensity passed over in going from the threshold intensity, E_0, to any higher intensity, E_n, is equal to the number of binomial factors on the right-hand side of the expression:

$$E_n = E_0(1 + \Delta E/E]_0)(1 + \Delta E/E]_1) \cdots (1 + \Delta E/E]_{n-1})$$

where $\Delta E/E]_0$ is the differential sensitivity at the threshold, $\Delta E/E]_1$ is the differential sensitivity at the end of the first step, etc. Fig. 1 shows the number of distinguishable steps as a function of intensity expressed in db above threshold for various frequencies.

Having these curves, consider the equal loudness measurements presented by W. A. Munson at the May, 1932 meeting of the Acoustical Society.[2] Equal loudness contours were determined by balancing pure tones of various frequencies against 1000 cps tones at levels 10, 20, 30, etc., db above the auditory threshold. Now, to investigate a possible relation between these equal loudness contours and the curves of Fig. 1, let it be assumed that a single such equal loudness contour has been ex-

[1] R. R. Riesz, *The Differential Intensity Sensitivity of the Ear for Pure Tones*, Phys. Rev. **31**, 867–875 (1928).

[2] W. A. Munson, *An Experimental Determination of the Equivalent Loudness of Pure Tones*, abstract, Program of Seventh Meeting of Acoustical Society of America, J. Acous. Soc. Am. **IV**, 7 (1932).

perimentally determined and let this contour be known as the reference contour. Then, let the hypothesis be proposed that two tones of different frequencies will sound equally loud when their intensities are such that the ratios of the number of distinguishable steps above threshold to the number of steps above threshold for the reference contour at the same frequency are equal for the two tones. Equal loudness contours calculated on the basis of this hypothesis are shown on Fig. 2, using Munson's fifth contour as the reference contour. The given data (represented

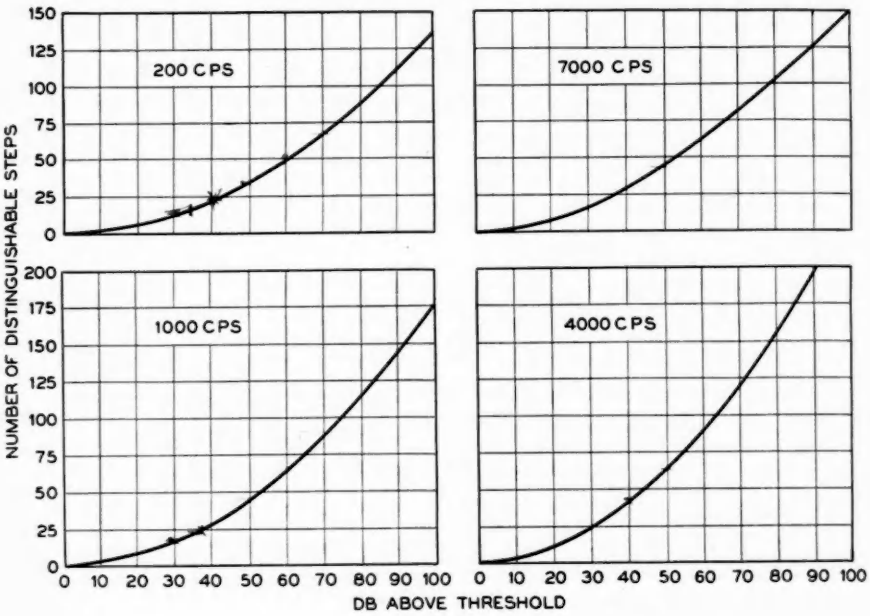


FIG. 1.

by circles) are the reference contour and the intercepts of the other contours at 1000 cps. The numbers indicated on Fig. 2 at 1000 cps are the ratios of the number of distinguishable steps above the auditory threshold to the number of steps above threshold for the reference contour. With the exception of the two upper contours the agreement between the calculated curves and the observed points (indicated by dots) is good. It is possible that the discrepancy at high intensities is due to nonlinear effects in the ear but it is interesting to note that at 4000 cps the upper calculated loundess contour is 21 db below the threshold of feeling while at 60 cps it is 43 db below the threshold of feeling. The discrepancy between the calculated contour and the observations may be due to a psychological effect caused by the fact that at the high frequencies the

calculated results would require the observer to select an intensity close to the threshold of feeling and due to the unpleasantness of so loud a sound he would compress his judgment and select a lower level. As the frequency is decreased this effect would play a less prominent part and the observed points would not be lowered. If this explanation is valid

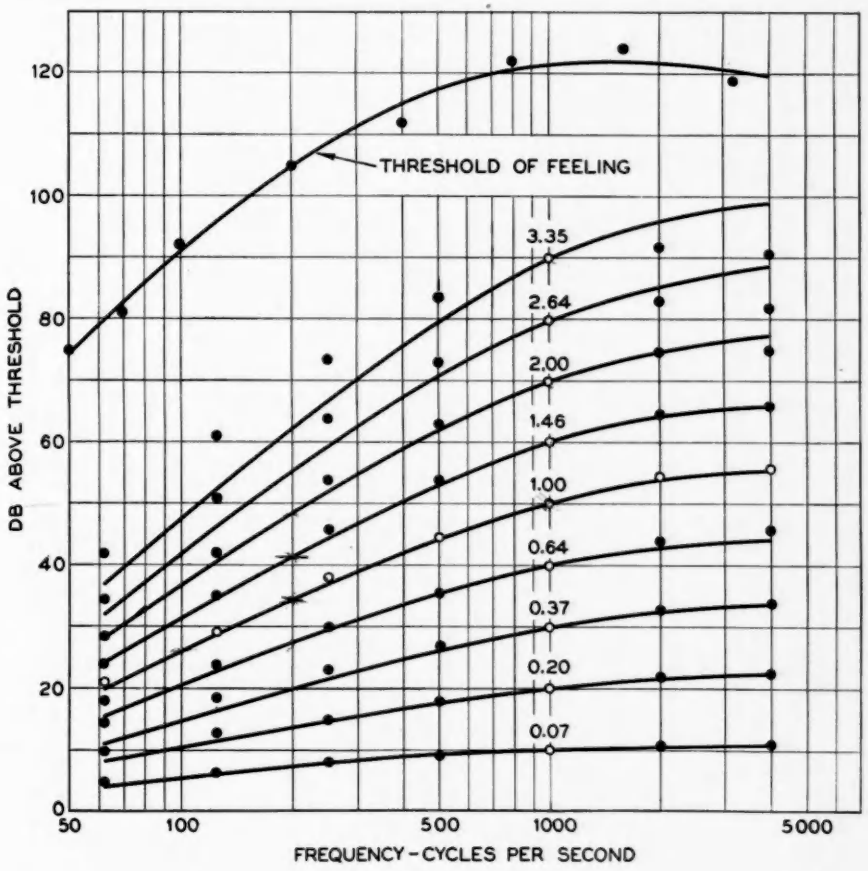


FIG. 2.

the observed equal loudness contour at high intensities would have its shape changed in the manner indicated, tending to approach parallelism with the curve of the threshold of feeling. However, independently of the validity of this explanation, the hypothesis proposed concerning the relationship between loudness and the minimum perceptible increment of intensity holds provided the intensities of none of the tones involved are higher than 40 db below the threshold of feeling.

Recently a number of workers[3] have made a series of loudness measurements of a character quite different from the equal loudness balances discussed above. These measurements are concerned with the estimation of the fractional reduction of loudness and are made by permitting an observer to listen to a sound at a known intensity and then asking him to judge at which intensity the sound (its quality remaining unchanged) has had its apparent loudness reduced or increased by a given fractional amount. There is considerable uncertainty as to the significance of measurements such as these and although the results of each experimenter are quite consistent they do not agree with one another by quite large amounts. The results of Geiger and Firestone indicate that the results are greatly dependent on experimental conditions. Until these discrepancies are satisfactorily accounted for or eliminated the discussion given below or any other attempt to arrive at a theoretical explanation of the phenomena cannot be considered final.

Bearing the above facts in mind, consider the data published by Laird and let an assumption be made as to the nature of the judgment made by an observer when he reduces the apparent loudness of a sound by a given amount and then let the calculations based on this assumption be compared with the experimental results. The differential sensitivity curves show that, for all frequencies, as the intensity is increased from the threshold value, $\Delta E/E$ decreases continuously, tending to approach a constant value at high intensities. If $\Delta E/E$ were constant throughout the entire range of intensities, the Weber-Fechner law, which states that the sensation is proportional to the logarithm of the physical stimulus, would be valid and loudness might be expected to be proportional to db above threshold. The fact that $\Delta E/E$ is not constant checks with the fact that Laird's observers did not, for instance, halve the number of db above threshold in selecting an intensity half as loud as the original. Now, let it be assumed that when an observer heard a tone, he had, from his past experience, some sort of mental image of the number of distinguishable steps of intensity between threshold and the level he heard, and when he selected a level of $1/n^{\text{th}}$ the apparent loudness of the original sound he selected a level $1/n^{\text{th}}$ the number of distinguishable steps above threshold that the original level was. Making these calculations for $1/n = \frac{1}{2}$ and for frequencies of 200,

[3] D. A. Laird, E. Taylor and H. H. Wille, Jr. *The Apparent Reduction of Loudness*, J. Acous. Soc. Am. **III**, 393 (1932). L. B. Ham and J. S. Parkinson, *Loudness and Intensity Relations*, J. Acous. Soc. Am. **III**, 511 (1932). P. H. Geiger and F. A. Firestone, *The Estimation of Fractional Loudness*, Program of Seventh Meeting of the Acoustical Society of America.

1000, 4000 and 7000 cps, the results are seen to show (Fig. 3) only a slight dependence upon frequency, even though the original curves shown on Fig. 1 vary considerably with frequency. This practical independence of frequency is exactly what Laird found experimentally, the results for 64, 256, 1024 and 4096 cps and for the buzzer tone of the Western Electric No. 3A Audiometer differing but slightly.

Making the same calculation for $1/n = \frac{1}{4}$, $\frac{3}{4}$ and 2 gives the curves shown on Fig. 4. The points indicated represent Laird's average data. The agreement between calculated and observed results is best for

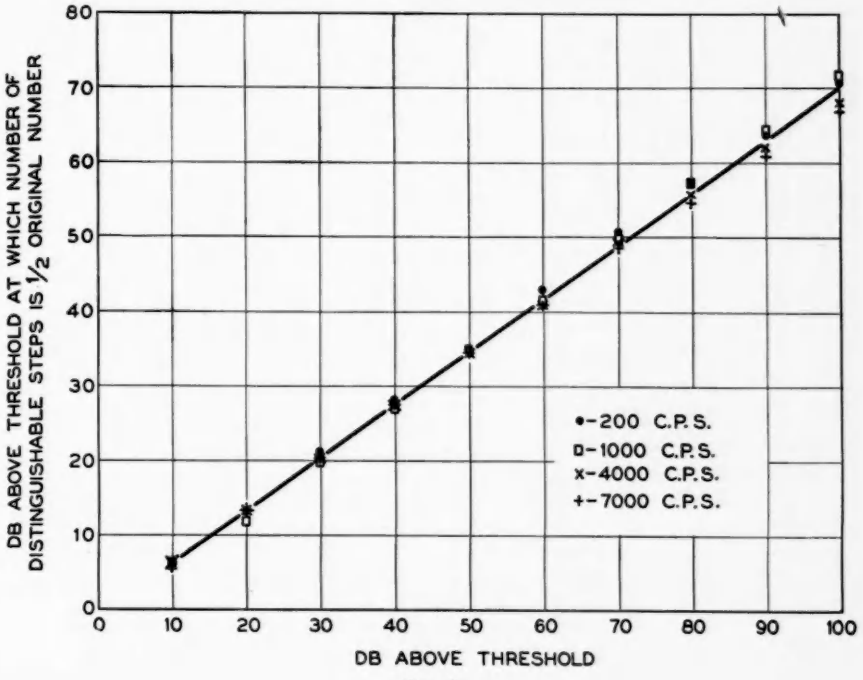


FIG. 3.

$1/n = \frac{1}{2}$ which probably would be the simplest judgment for the observer to make. The large discrepancy between calculated and observed results at high intensities for $n = 2$ may be due to the same cause as that suggested for the discrepancy between calculated and observed equal loudness contours at high intensities. These results show that as far as the data of Laird are concerned it is possible to establish an approximate relationship between the apparent reduction of loudness measurements and the minimum perceptible increment of intensity, which is a property of the ear capable of precise measurement and suggest that

for measurements of this kind the curve of number of distinguishable steps *vs.* db above threshold may be useful as a calibration curve for estimating loudness.

In conclusion, it must be emphasized that although Laird's acoustically inexperienced observers, when asked to select a sound $1/n^{th}$ as

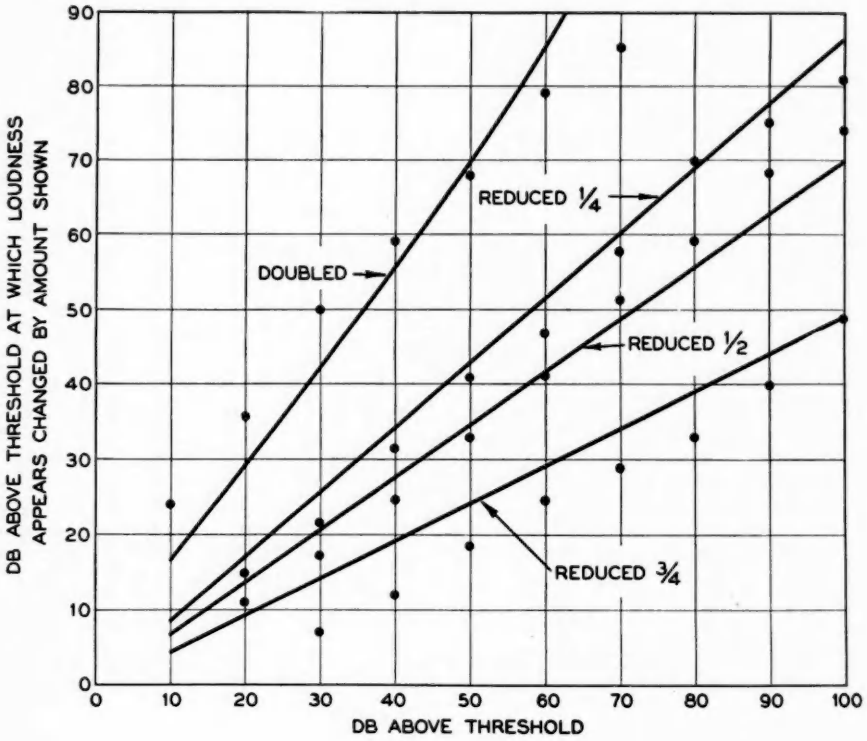


FIG. 4.

loud as a given sound of the same quality, selected a level closely corresponding to $1/n^{th}$ the number of distinguishable steps above threshold of the original sound, it is not apparent at the present time that this result can be generalized or accepted as the final explanation of the phenomena. Laird reported failure to get consistent results using acoustically trained observers. Also the results of Ham and Parkinson and of Geiger and Firestone are not in agreement with the above rule.

USE OF PRESSURE GRADIENT MICROPHONES FOR ACOUSTICAL MEASUREMENTS

By Irving Wolff and Frank Massa
Research Division, RCA Victor Company, Inc.

Abstract

The operation of the pressure gradient microphone is compared with that of the pressure microphone It is shown that the pressure gradient microphone may be used to measure particle velocity in a sound wave. The advantages of the pressure gradient microphone in making loudspeaker measurements, particularly outdoors, are pointed out and experimental data are given for some arrangements which were tried out. The characteristics of the distribution of particle velocity in a complex sound field are studied theoretically and experimentally with a ribbon microphone. A method is described for measuring the energy density in a sound field and some measurements which were taken in complex sound fields in rooms are discussed. It is shown that a combination of three pressure gradient or velocity microphones with a pressure microphone, placed adjacent to each other, may be equivalent in eliminating interference patterns to four pressure microphones placed at distances large compared to the wavelength and with random distribution. A microphone for measuring energy flow in a sound field is described.

Introduction

Until very recently the condenser microphone has been the most commonly used microphone for acoustical measurements. It has been generally selected because of its comparatively flat frequency response, freedom from disturbing sounds when in good operation, and constancy of calibration. It has been realized, however, that the condenser microphone possesses characteristics which introduce errors when it is used for certain acoustical measurements. In the first place, being a pressure actuated device, it is sensitive to sound coming from all directions and cannot discriminate against any undesirable component at the lower frequencies where its size is small compared to the wave-length. Secondly, the construction of the condenser microphone is such that distortion is introduced in the sound field in which it is being used at frequencies above 4000 cycles. The distortion is introduced both by the fact that at the higher frequencies, the diaphragm acts as an infinite wall, causing the effective pressure at its surface to be doubled; and also, the air cavity which exists in front of the diaphragm introduces resonance, causing the sensitivity of the microphone (to sound travelling at normal incidence to the diaphragm) to be increased by over 100 percent for even small microphones which have been particularly constructed for measurement work.

A type of ribbon microphone has been described by Olson[1] which eliminates many of the disadvantages of the condenser microphone, and

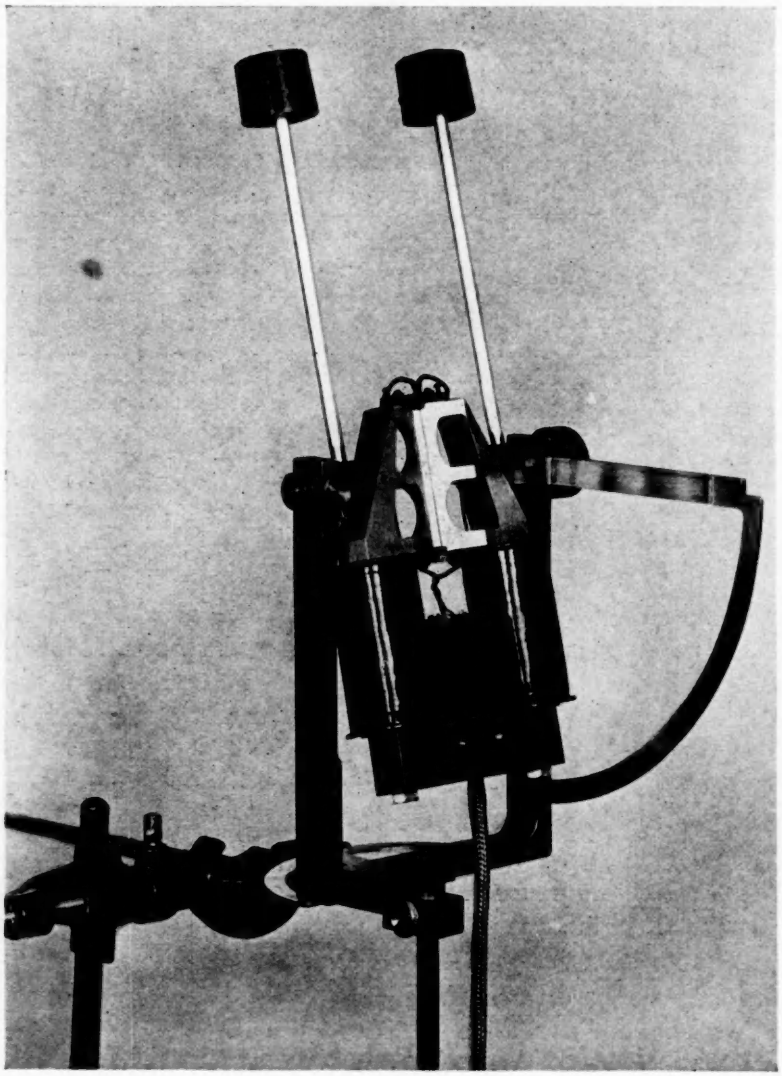


FIG. 1.

which, because of its directional characteristics and means of operation, may be used alone and in conjunction with a pressure operated micro-

[1] Olson, *Mass Controlled Electrodynamic Microphones, The Ribbon Microphone,* J. Acous. Soc. Am. **III,** 56 (1931); *The Ribbon Microphone,* J. Soc. Mot. Pic. Eng. **XVI,** 695 (1931).

phone for many measurements which cannot be made with the condenser microphone or which are made with difficulty.

The condenser microphone is actuated by the difference between sound wave pressure and atmospheric pressure. It has been shown by Olson[1] that the ventilated ribbon microphone is actuated by the difference between sound wave pressure on the front and rear sides. When the wave-length is long compared to the path from front to rear this difference in pressure is proportional to the space variation in alternating pressure or pressure gradient at the point where the microphone is located.

The ribbon microphone that was developed by the RCA Victor Company for acoustical measurements is shown in Fig. 1. The open structure in the vicinity of the ribbon is sufficient to prevent the microphone from reacting on the sound field in a way to introduce frequency distortion at the ribbon. This was determined experimentally by calibrating the unit with a Rayleigh disk, and showed a very smooth response (variations less than 1 db), flat up to 1500 cycles and then a gradual drop amounting to 6 db at 15,000 cycles. This type of frequency characteristic is very easily compensated for in the associated amplifier.

The microphone as shown in Fig. 1 is mounted on a fixture which permits rotation of the unit about a vertical and horizontal axis. A transformer is mounted on the magnetic structure which steps up the ribbon impedance to 500 ohms, permitting the use of long leads to the amplifier.

PRESSURE GRADIENT MICROPHONE MEASURES PARTICLE VELOCITY IN SOUND FIELD

It is of interest to note that the pressure gradient microphone measures particle velocity in a sound field. When steady state conditions are reached in a sound field, the following relations will hold, by using the symbols: p = sound pressure; ρ_0 = density of the medium; ϕ = velocity potential; $\nabla = \partial/\partial x + \partial/\partial y + \partial/\partial z$; u = particle velocity; ∇p = Pressure gradient;

$$\phi = F(x, y, z) \cos [\omega t + f(x, y, z)] \tag{1}$$

$$d\phi/dt = -\omega F(x, y, z) \cos [(\omega t - \pi/2) + f(x, y, z)] \tag{2}$$

since

$$p = -\rho_0 d\phi/dt \tag{3}$$

$$u = \nabla\phi \tag{4}$$

$$\nabla p = -\rho_0 \nabla d\phi/dt. \tag{5}$$

We can substitute (1) and (2) in (4) and (5) getting

$$u = \nabla F(x, y, z) \cos [\omega t + f(x, y, z)] \tag{6}$$

$$\nabla p = \rho_0 \omega \nabla F(x, y, z) \cos [(\omega t - \pi/2) + f(x, y, z)] \tag{7}$$

showing that with the exception of a phase shift of $\pi/2$, the space distribution of pressure gradient is the same as that of particle velocity.

LOUDSPEAKER MEASUREMENTS

Possibly one of the most practical applications of a microphone as a means for measuring sound intensity has been in the determination of frequency-response characteristics of loudspeakers. The disadvantage of using a condenser microphone has already been mentioned; *viz.*, fre-

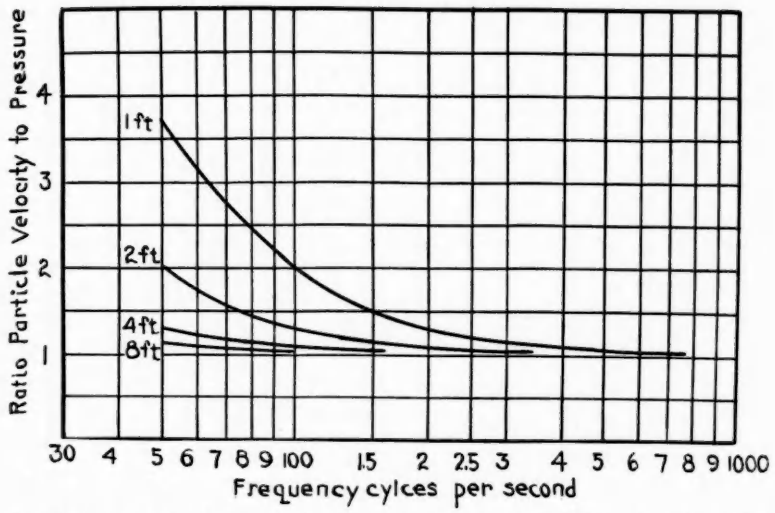


FIG. 2. *Showing accentuation of the low-frequency response in a ribbon microphone at various distances from a point source of sound.*

quency distortion at the high frequencies and no discrimination against undesirable reflected sound. The ribbon microphone, although free from the above disadvantages, possesses another possible type of frequency distortion which must be guarded against. At low frequencies there is a tendency to give false readings when used too close to the source of sound.

Assuming a spherical wave diverging from a source, calculation shows that a measurement of pressure gradient or velocity gives a value which is too high in the ratio $(1+c^2/\omega^2 r^2)^{1/2}$ where $\omega = 2\pi \times$ frequency, r is the distance from the source, and c the velocity of propagation of sound in

the medium. Fig. 2 shows the error introduced as a function of frequency at various distances from a point source of sound. This difficulty can be easily overcome by choosing a distance which will not introduce error at the lowest frequency concerned. If the distance is somewhat too short, so that a small error is introduced, the formula given above may be used to obtain an approximate correction.

The ribbon microphone is very adaptable for loudspeaker measurements out of doors. This results from the particular directional characteristic of the ribbon microphone, or in fact of any pressure gradient microphone. As has been shown theoretically and experimentally by

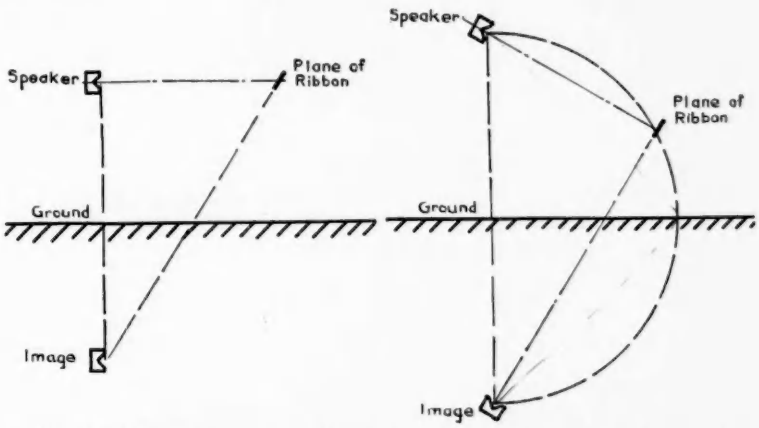


FIG. 3. *Showing two methods for arranging loudspeaker and ribbon microphone so that ground reflections will not be introduced in the data.*

Olson[1] the directional characteristic at all frequencies is very nearly that shown in Fig. 15. Reference to the figure shows that there is a plane of zero response corresponding to the plane of the ribbon while the response in all other directions is proportional to the cosine of the angle made between the direction and the perpendicular to that plane.

Usually the only reflection which is of any consequence when making loudspeaker measurements out of doors is that from the ground, and this can be prevented from affecting the microphone reading by arranging the units as shown in Fig. 3, with the plane of zero response of the ribbon in the direction of the reflected wave. The completeness with which the ground reflections are ruled out is indicated in Fig. 4. The solid curve was taken with a condenser microphone placed 10 ft. from a loudspeaker, both speaker and microphone being $6\frac{1}{2}$ ft. above the ground. The dotted curve was taken by using a ribbon microphone in-

stead of the condenser, with the axis tipped to eliminate the ground reflections.

An interesting group of curves is shown in Fig. 5 which indicate, quantitatively, the extent to which ground reflection errors are neutralized by using a ribbon microphone. Curve (a) shows the theoretical deviation in sound pressure due to perfect ground reflections at a point 10 ft. from a sound source which is $6\frac{1}{2}$ ft. from the ground. A spherical distribution of sound is assumed at the source and 100 percent reflection

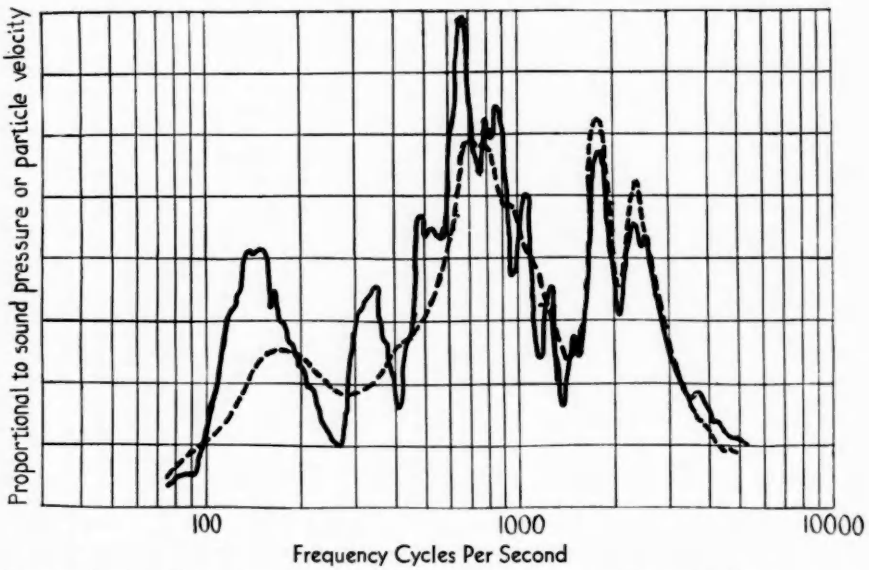


FIG. 4. *Showing elimination of ground reflection peaks by use of ribbon microphone.* - - - - -, *curve taken with ribbon microphone (plane of ribbon in line with speaker image).* ———, *curve taken with condenser microphone (corrected for pressure doubling and cavity resonance). Speaker and microphones* $6\frac{1}{2}$ *feet above ground. Distance between microphones and speaker 10 feet.*

from the ground. The actual error introduced in a loudspeaker response curve taken with a condenser microphone is shown in curve (b). The deviation from the theoretical case at the high frequencies is normal since the sound distribution at these frequencies becomes more directional, resulting in a smaller component reaching the ground for reflection. Curve (c) shows the error introduced when a ribbon microphone was substituted for the condenser. The plane of the ribbon was vertical and the error introduced is less than in the condenser microphone, due to the fact that the reflected component striking the ribbon is effective only as the cosine of the angle which it makes with the normal to the ribbon. Curve (d) shows the error completely removed by tilting the

plane of the ribbon so that the reflected component arrives in a direction parallel with the plane of the ribbon.

When we take loudspeaker measurements indoors in a fairly "dead" room, the first reflections are usually the most serious. Generally, the room is fairly large so that the loudspeaker and microphone are at a large distance from the walls. In this case, the most serious reflection

(a) Theoretical deviation in sound pressure at the microphone caused by ground reflection. Point source of sound and 100% reflection from ground assumed.

(b) Actual error introduced in a loudspeaker frequency characteristic due to ground reflection, by using a condenser microphone.

(c) Actual error introduced by using a ribbon microphone with the plane of the ribbon vertical.

(d) Actual error introduced by using a ribbon microphone with the plane of the ribbon in line with speaker image.

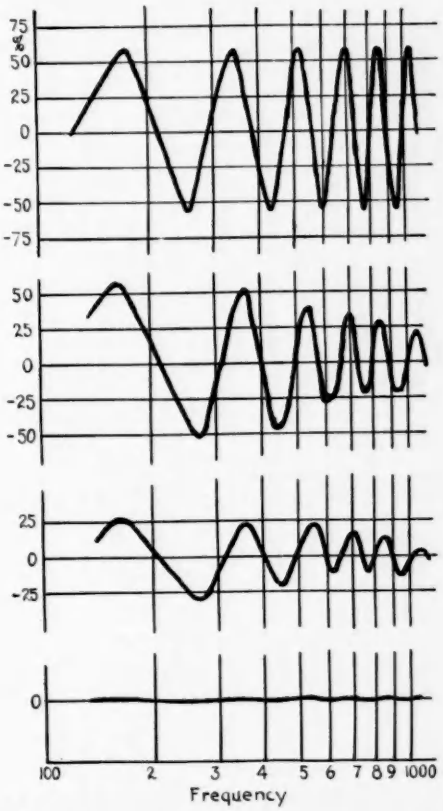


FIG. 5. *Showing the amount of error introduced in a loudspeaker frequency-response characteristic by using different methods of measurement. Distance of speaker and microphone from ground 6½ feet. Distance from speaker to microphone 10 feet.*

is that from the floor and the microphone can be placed to discriminate against this component. In special cases, where the loudspeaker and microphone can always be placed in the same predetermined position, it is possible to design the room with sloping walls and ceiling, such that all the more important reflections will lie in the plane of the ribbon, resulting in only the direct sound actuating the microphone.

It can be concluded that the superiority of the pressure gradient type

of microphone for loudspeaker measurements lies, firstly, in the fact that no distortion is introduced by the microphone at the high frequencies, and, secondly, that the microphone can be arranged to discriminate against undesirable reflections.

MEASUREMENT OF SOUND FIELD IN A ROOM

In rooms where diffuse sound field conditions do not exist, it is often of interest to determine the amount of energy which is transmitted in different directions across the room. The use that can be made of the pressure gradient microphone to do this is illustrated by Figs. 6 and 7,

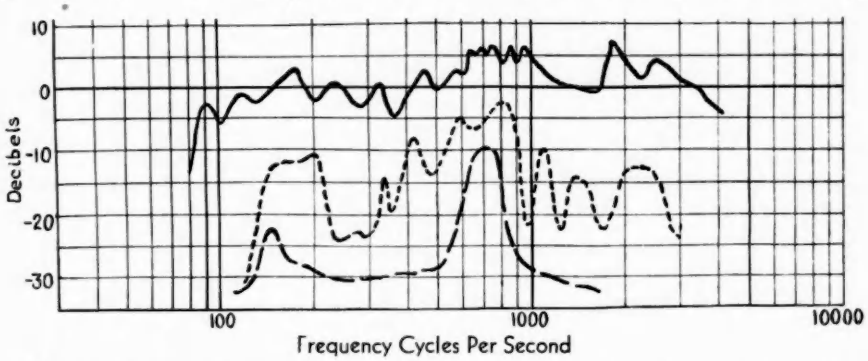


FIG. 6. *Application of ribbon microphone to indicate the amount of sound reflected from various wall surfaces in a fairly "dead" room.* ———, *plane of ribbon normal to loudspeaker axis.* - - - -, *plane of ribbon in loudspeaker axis and parallel to floor (distance between floor and ceiling unequal).* — — —, *plane of ribbon in loudspeaker axis and parallel to floor (distance between floor and ceiling equal). Room 20 feet square×10 feet high. All surfaces 4 inch mineral wool.*

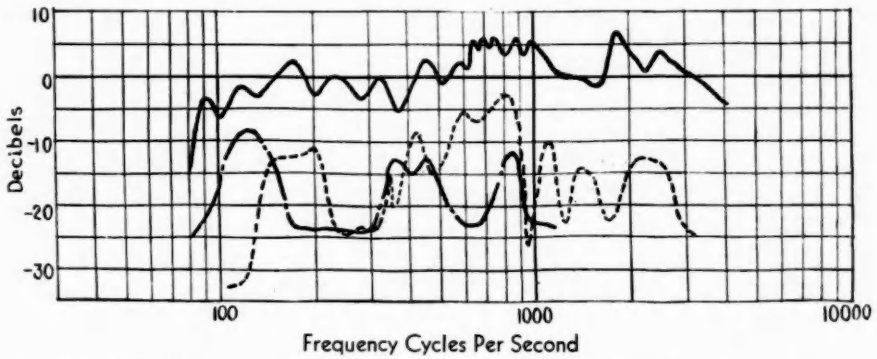


FIG. 7. *Application of ribbon microphone to indicate the amount of sound reflected from various wall surfaces in a fairly "dead" room.* ———, *plane of ribbon normal to loudspeaker axis.* —· — · —, *plane of ribbon in loudspeaker axis and parallel to side wall.* - - - - -, *plane of ribbon in loudspeaker axis and parallel to floor (distance between floor and ceiling unequal). Room 20 feet square×10 feet high. All surfaces 4 inch mineral wool.*

which show the sound being transmitted along various axes of a fairly dead room. In a "live" room, the variations of sound transmission along various axes would naturally not have been as noticeable as indicated above. The method of measurement used for Figs. 6 and 7 may be applied to the study of sound reflections from various types of walls.

MEASUREMENT OF PARTICLE VELOCITY IN A SOUND FIELD

As shown above (Eqs. (1) to (7)) the pressure gradient microphone will measure the particle velocity in a sound wave. The instantaneous value of the particle velocity is a simple vector quantity, which can be projected on any axis, obeys the cosine law and can be added vectorially, $v_\theta = v \cos \theta$, where θ is the angle between any direction and the direction in which the particle is moving, v is the instantaneous particle speed and v_θ is the projection of this instantaneous speed in the direction θ. The assumption should not be made on the basis that the velocity measured by the microphone, which depending on the type of electrical detector which is used, is either mean, square, average or maximum value, is also a vector of this type. Because of the probability of time phase differences, care must always be taken in determining the particle velocity due to a number of waves in a specified direction to add instantaneous values vectorially and not the maximum, mean square or average values given by the indicating instrument.

In plane or spherical progressive sound waves the value measured by the microphone can be taken as a simple vector quantity and may be predicted in any direction by the cosine law since $v_\theta = V \cos \theta \cos \omega t$ and $(v_\theta)_{max} = V \cos \theta$.

A simple illustration to show that the measured velocity is not a simple vector quantity in a sound field in which interference takes place is obtained by studying the conditions when two equal plane wave trains travelling at right angles in the same plane intersect. Points in the region of intersection may be found in which the time phases of the two wave trains differ by $\pi/2$. If the instantaneous velocity in the one train is $v_1 = V \cos \theta \cos \omega t$, that in the other is $v_2 = V \sin \theta \sin \omega t$, where v_1 and v_2 are the components of the instantaneous velocity in the direction θ. The sum in direction θ is

$$v_\theta = V(\sin \theta \sin \omega t + \cos \theta \cos \omega t); \; v_\theta = V \cos (\theta + \omega t)$$

showing that the maximum, average or mean square values of the velocity are independent of the direction and the microphone should therefore give a reading on the electrical indicating instrument independent of direction in the plane of the two waves.

The particle velocity at these particular points in space is of the same nature as the field due to the electric or magnetic vectors in a circularly polarized light wave or like the rotating field in certain types of electrical generating or motor machinery. By choosing the magnitudes, directions and phase relations of the interfering waves properly, any form of rotation corresponding to elliptically polarized light may be obtained.

A ribbon microphone was mounted on a double swivel arrangement so that its plane could be rotated into any position. The readings taken in a plane progressive wave have corresponded to those to be expected for the vector projection on a perpendicular to the plane of the ribbon while those taken in a standing wave system show the elliptical and, in very special cases, circular rotating vector to be expected from the analysis.

In order to obtain the absolute value of particle velocity at a point, it is necessary to measure three right angle components and then add them vectorially. This may be accomplished by having three ribbon microphones mutually perpendicular with the distances between them small compared to the wave-length of the sound being measured. The outputs from each microphone must be squared and added, the result being the square of the particle velocity which is proportional to the kinetic energy in the sound field at the point. An alternative method is to make three independent readings at the same point with the microphone facing along right angle axes. This latter method was adopted in our measurements as it was considered the more expedient at the time.

In order to obtain a check on the operation of the ribbon microphone, readings were taken of the components along different sets of right-angle axes in the same plane in order to see if the vector sum for any pair of axes would be the same. Some typical data are shown in Table I,

TABLE I. *Showing the agreement of particle velocity at a point by measuring different sets of right-angle components in the same plane.*

Frequency	0°	90°		30°	120°		45°	135°		75°	165°	
150~	99	18	[117]	55	61	[116]	32	84	[116]	10	109	[119]
400~	170	10	[180]	124	57	[181]	83	103	[186]	16	166	[182]

the data given being proportional to the square of the particle velocity. The bracketed figures are the sum of the squares of the component velocities measured along the axes indicated on the chart and the agreement is well within the probable error of the measurements.

ENERGY DENSITY MEASUREMENTS

The energy density in a sound field is both kinetic and potential, and in a field in which standing waves are present may at some points be entirely potential and at others entirely kinetic, or consist of any combination of the two. The potential energy is proportional to the square of the pressure, while the kinetic energy is determined by the square of the air particle velocity. One method of measuring the energy density directly is to have one microphone which will measure the potential

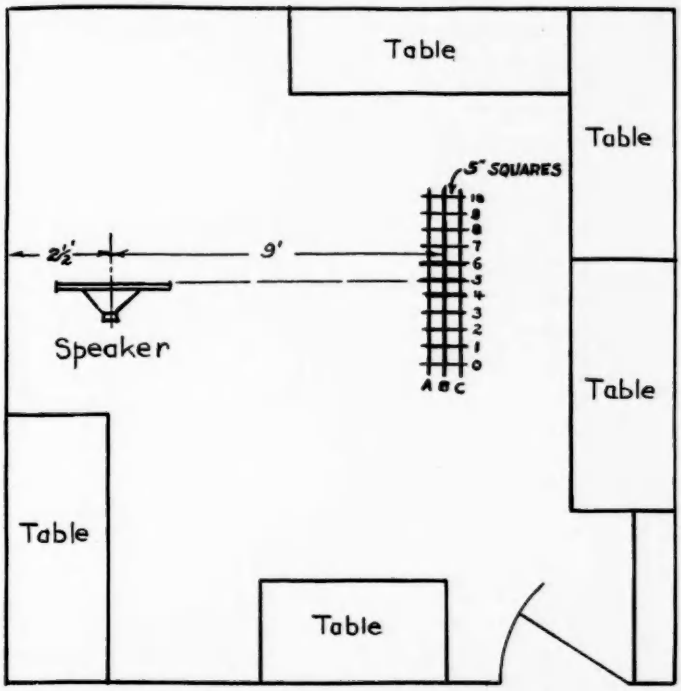


FIG. 8. *Showing arrangement for energy measurements at a set of points within a closed room. Room 16×16×16 feet. Celotex on walls and ceiling.*

energy and another which will measure the kinetic energy. It is well known that the condenser microphone measures the pressure in the sound wave, and, when small compared to the wave-length, exerts negligible influence on the sound field. It, or some other similar microphone, may therefore be used to measure potential energy. It was shown above how the ribbon microphone was used to measure particle velocity in a sound field; therefore, with the aid of both types of microphones the energy density at any point can be measured.

An experimental investigation of the distribution of energy density

within a closed room, in which a constant sound signal was maintained, was conducted in order to learn how the variations of total energy from point to point compared with the variations in either the potential or kinetic energy. The investigation was conducted in a room approximating a 16 ft. cube which had a hard wood floor and $\frac{1}{2}$ inch celotex on the walls and ceiling. Several tables lined the walls of the room, and the acoustical characteristic of the room was about equal to that of an average living room.

For the first stage in the investigation, a set of points were surveyed, as shown on Fig. 8. The speaker and microphones were placed 3 ft. from

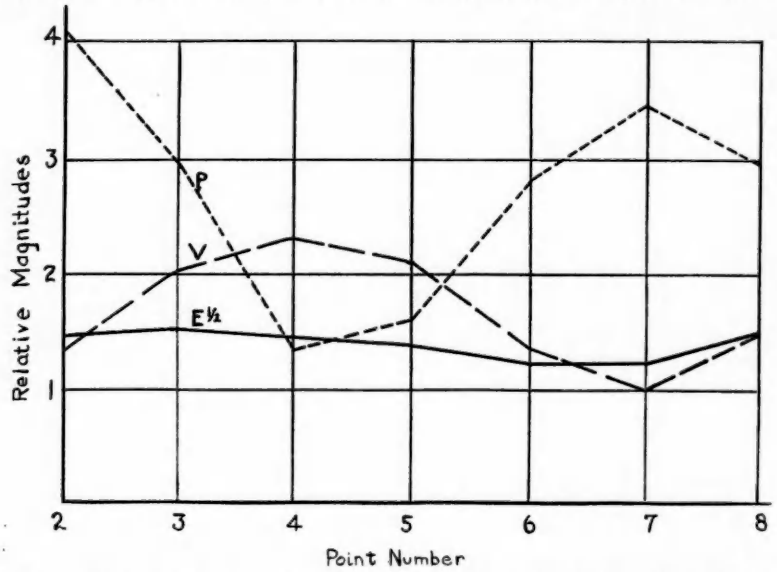


FIG. 9. *Showing variations in P, V and $E^{1/2}$ along row A of the points shown in Fig. 8 Frequency=290 cycles.*

the floor. The variation of the pressure, particle velocity and square root of energy along the points of row *A* of Fig. 8 is shown in Fig. 9. In Fig. 10 is shown another set of points that were surveyed. Figs. 11 to 14 show the variations in pressure, particle velocity, and square root of energy along the points on arcs *B* and *G* with two different sound sources; *viz.*, a 6 inch cone radiating from one side and then a 12 inch cone mounted in a 3 ft. square baffle. The curves on Figs. 11 to 14 are typical of all the arcs. As was to be expected, as the distance from the speaker increased the variations from point to point along the arcs became more violent because of the direct radiation from the cone being less effective at the greater distance.

In order to have a somewhat more quantitative measure of the variations in these quantities, the data covering the points on Fig. 10 were taken and analyzed by finding the average deviation from the mean along each arc. This was done for each velocity component, total velocity, pressure and energy. In addition to the points shown on Fig. 10, several observations were made keeping the speaker and microphone always 10 ft. apart in the room and moving them both around at ran-

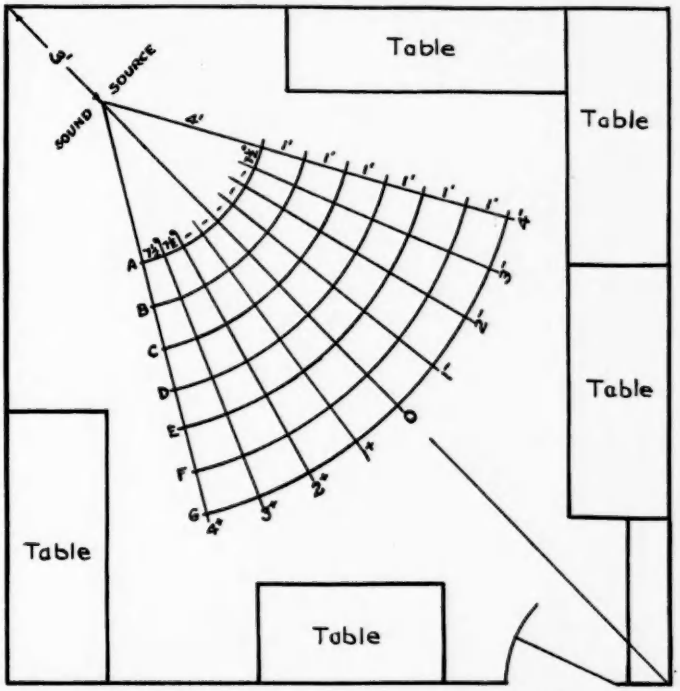


FIG. 10. *Showing arrangement for energy measurements at a set of points within a closed room. Room 16×16×16 feet. Celotex on walls and ceiling.*

dom positions within the room. The data on these points were also averaged, and in Table II is summaried the average deviations from the mean for all the above cases.

From Table II, it can be seen that, in general, the deviations in the pressure and two of the three velocity components have the same order of magnitude. The velocity component in which the ribbon has its plane normal to the speaker axis does not vary as much, which is to be expected since the ribbon will favor the direct sound from the loudspeaker because of its directional characteristic. A considerable reduction in the variations from point to point can be seen in the case of total particle

velocity measurements (column 5). Total energy distribution is still more constant, as seen in column 6.

TABLE II. *Showing average deviations from the mean for different components of sound energy measurements within a closed room in which a constant sound source is maintained.*

	% P or $(PE)^{1/2}$	% V_1 Plane of ribbon in LS axis	% V_2 Plane of ribbon parallel to floor	% V_3 Plane of ribbon normal to LS axis	% V_0 or $(KE)^{1/2}$	% $E^{1/2}$
12″ cone on 3′×3′ baffle	38	36	31	26	17	15
6″ cone in box	28	36	28	27	16	11
Loudspeaker and microphone 10′ apart and moved about	51	51	49	33	22	21

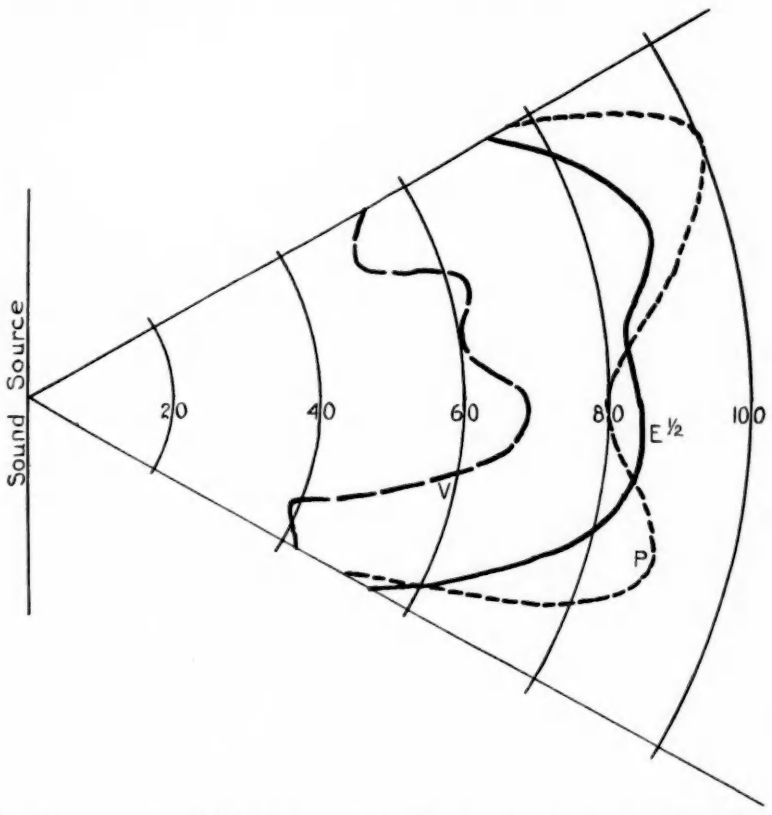


FIG. 11. *Showing P, V and $E^{1/2}$ along arc B 5 feet from speaker, 6 inch cone radiating from one side. $f=290\sim$.*

It will be noted that the $(KE)^{1/2}$ shows considerably less deviation than the $(PE)^{1/2}$, due to the fact that the $(KE)^{1/2}$ is made up of three independent velocity components. The improvement in the $E^{1/2}$ over the $(KE)^{1/2}$ is about what would be expected from the addition of an extra microphone and does not show the improvement which might be hoped for from some expected correlation between maxima of KE and

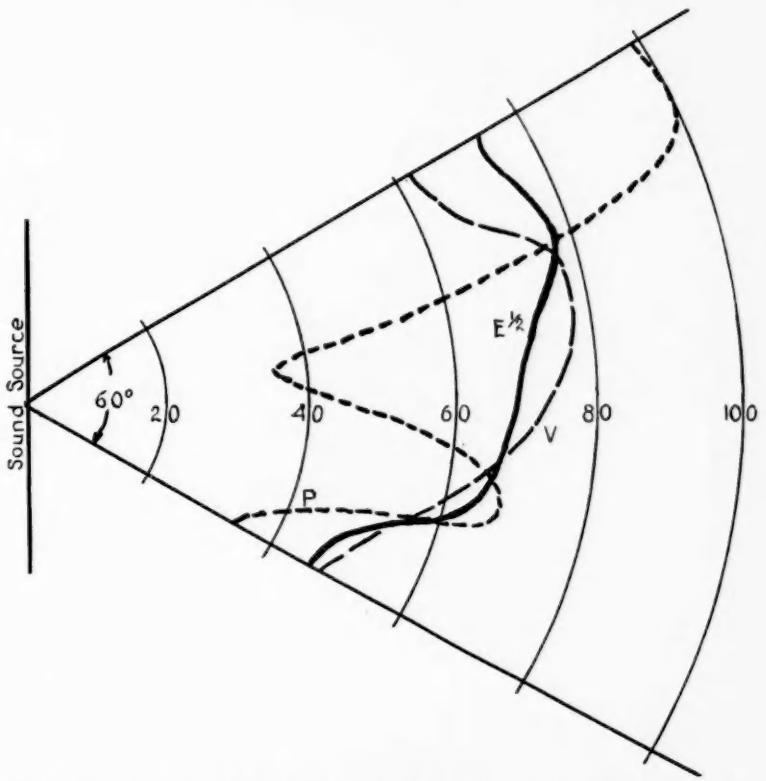


FIG. 12. *Showing P, V, and $E^{1/2}$ along arc B 5 feet from speaker. 12 inch cone in 3×3 feet baffle. $f=290\sim$.*

minima of PE or *vice versa*. Practically, the use of three pressure gradient microphones with their axes mutually perpendicular plus a pressure microphone obtains the effect of averaging the readings of four pressure microphones placed at random distances from each other and several wave-lengths apart, so that there is a random correlation between the readings of the microphone. The saving in space and simplicity of mounting obtained by using the pressure gradient microphone is a real advantage.

In making measurements of the kind described with four separate microphones, they must of course be used with separate amplifiers so that the outputs are not added until after they are detected. As a practical matter, the four microphones can feed into amplifiers, each one of which is terminated by a thermocouple, the outputs of all the thermocouples being in series so as to give a single output deflection on the meter which is porportional to the energy.

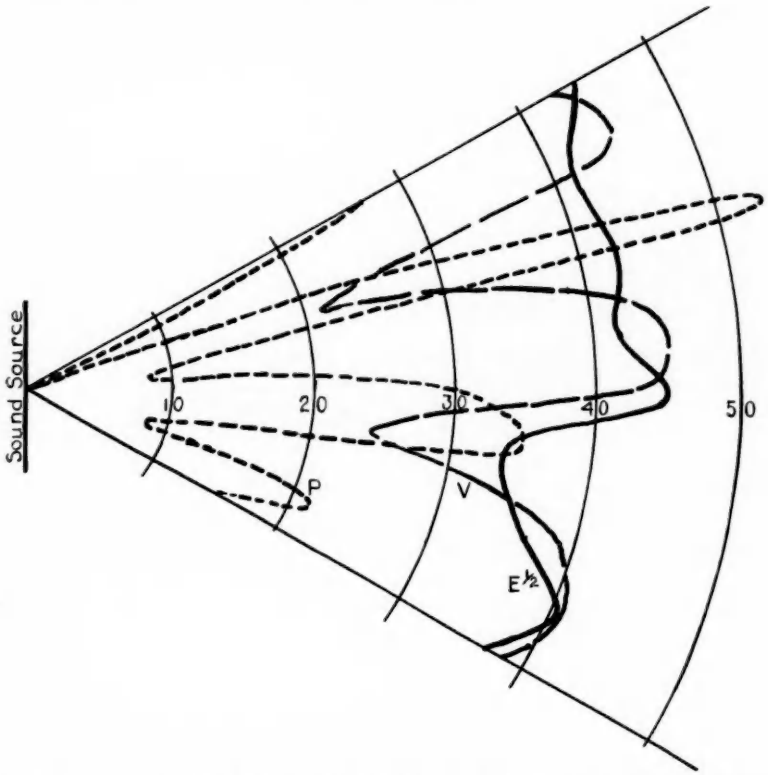


FIG. 13. *Showing P, V and $E^{1/2}$ along arc G 10 feet from speaker. 6 inch cone radiating from one side. $f=290\sim$.*

RIBBON MICROPHONE ROTATING ON ITS AXIS EQUIVALENT TO TWO MICROPHONES IN CANCELLING STANDING WAVE PATTERNS IN A ROOM

The difficulty of obtaining a true picture of the sound in a closed room because of irregularities due to interference is well known. A system has been suggested in the preceding section which requires three or four microphones, each having a separate amplifier, if a continuous measurement is to be made. A single amplifier may of course be used if three or

four readings may be taken, one after the other, and added subsequently. It is a matter of interest that a single ribbon microphone, rotated on its own axis so that it occupies very little more space than the microphone which is standing still, can be used to take continuous readings equivalent to that obtained by means of two microphones. This is possible as evident from the following considerations. It has been shown that readings taken with a ribbon microphone where the microphone is

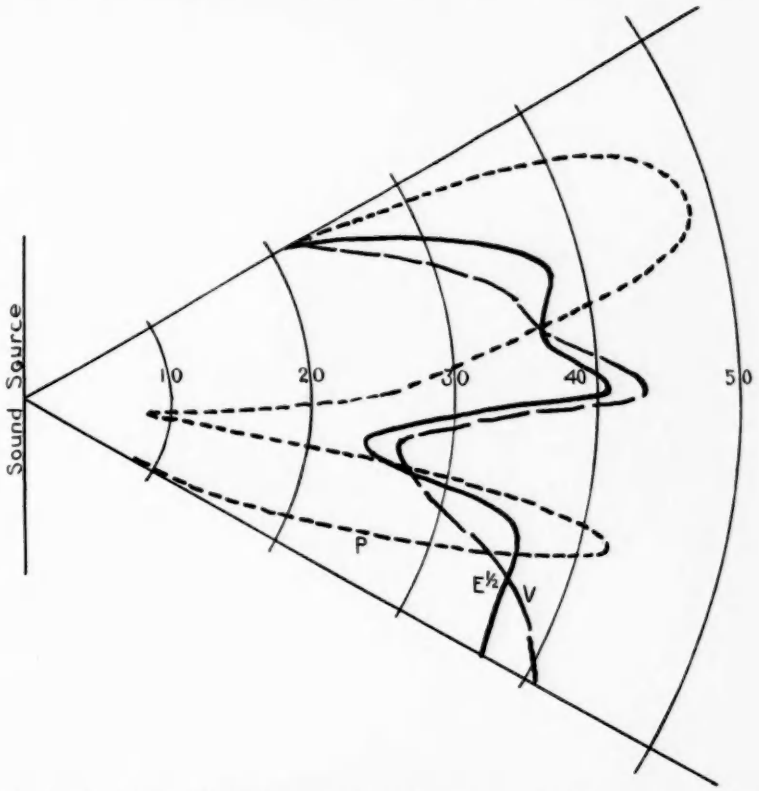


FIG. 14. *Showing P, V, and $E^{1/2}$ along arc G 10 feet from speaker. 12 inch cone in 3×3 feet baffle. f=290∼.*

rotated into positions where the plane of the ribbon differs by 90°, lead readings which are equivalent to two separate microphones disposed at random. It has also been shown that when rotated about an axis, all pairs of 90° positions having their outputs squared and added give the same sum. If the microphone is rotated continuously about an axis and red into an amplifier with a square law detector, the readings sweep through all pairs of 90° positions and if the microphone is rotated rap-

idly enough in comparison with the speed of response of the thermocouple and meter, a continuous reading is obtained which is equivalent to that of two microphones placed at random.

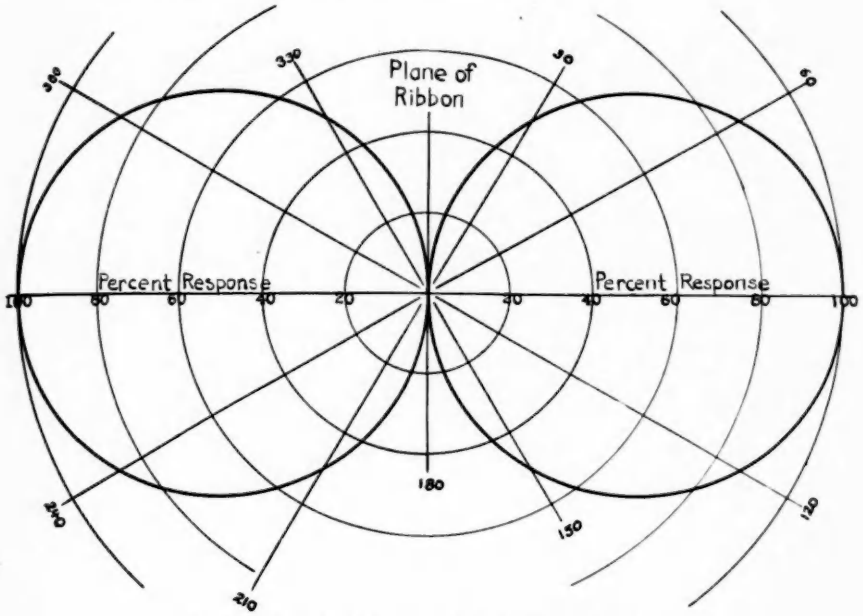


FIG. 15. *Directional characteristic of ribbon microphone.*

ENERGY FLOW MICROPHONE

Another use for a pressure gradient microphone in conjunction with a pressure microphone has been suggested by H. F. Olson as a means for measuring energy flow in a sound field. The energy flow in a given direction can be determined by taking the time average of the instantaneous product of the pressure and the velocity in the direction in which the flow is desired. Since the pressure gradient microphone measures velocity, it can be combined with a pressure microphone by using electrical circuits in which the product of the two outputs are taken to give a reading which is proportional to the energy flow perpendicular to the axis of pickup of the velocity microphone. Care must be taken to compensate for any phase shift which takes place in going from the acoustic wave to the electrical wave, since the amount of transmission of energy depends very largely on the relation of the velocity and pressure components in the sound wave.

CALIBRATION OF CONDENSER-MICROPHONES FOR SOUNDMETERS[1]

By E. J. Abbott
Department of Engineering Research, University of Michigan

The existing literature on microphone calibration consists largely of description of the various methods which may be used to compute the pressure applied to the diaphragm of the microphone to be calibrated. Very little has been published concerning the constancy of microphones, and the accuracy with which their calibration can be determined. These points are of considerable importance to those who use soundmeters because the microphone represents the only part of the instrument which cannot be calibrated readily and which is ordinarily depended upon to maintain its calibration over considerable periods of time.

In many cases it is desirable if not essential to measure sound to an accuracy which at first thought appears absurd. Ordinarily changes of less than a decibel or two are hardly worth considering, but in many cases of machinery noise reduction one is forced to work with changes much less than this. It is well known that when several sounds of approximately the same loudness are present, removing any one of them usually changes the total energy by less than a decibel. This is almost invariably the case in machine noise problems, and one is forced to study changes which in themselves produce a negligible effect, but which when properly combined with similar changes produce a marked effect. Another example was found by the author[2] in which the sounds were very loud so that the minimum perceptible difference in loudness was of the order of 0.2 to 0.3 db instead of approximately 1 db as at ordinary levels. Consequently the effect of a given change was three or four times as great, which required a corresponding increase in the accuracy of measurement. This was the starting point for the work here described.

The following program was undertaken in order to determine the absolute calibration of the microphone used in taking these measurements, and to provide an accurate reference standard for our laboratory for future work. A group of three thermophones was constructed at this laboratory, and three microphones calibrated on them. Next a pair of these microphones was sent to the Bell Telephone Laboratories for their calibration after which it was proposed to recheck them here. It was

[1] Presented before the Acoustical Society of America, May 2, 1932.

[2] *Noise Specifications for Large Reduction Gears in Terms of Physical Units*, J. Acous. Soc. Am. **III**, 445 (1932).

also planned to compare calibrations with thermophones filled with air and with hydrogen. This program has been largely carried out, but the desired results were not entirely obtained for various reasons to be outlined. The principal difficulty seemed to be that the experiment was more of a test of the constancy of the microphones than a test of the method of calibration

METHOD

The method of calibration which appeared to be most convenient and suitable for the purpose was the thermophone. In this case we were not so greatly concerned with just which correction gave the most accurate

TABLE I. *Factors used in the Wente thermophone formula.*

Symbol		Name	Typical Values: Hydrogen		Air
		Handbook Values			
1	C_p	spec. heat of gas at const. press.	3.41		0.237
2	ρ_0	density of gas at 0°C and 76 cm	8.99×10^{-5}		129.3×10^{-5}
3	K_0	therm. cond. of gas at 0°C and 76 cm	41.6×10^{-5}		5.68×10^{-5}
4	k	ratio of spec. heats of gas		1.41	
5	C_p'	spec. heat of stip		0.0355	
*6	ρ_0'	density of strip g per cc		17.95	
*7	K_0'	therm. cond. of strip at 0°C		0.35	
		Measured Values			
8	α_r	therm. coef. resistance of strip		0.0015	
9	T_r	room temp. (°K)		297	
10	P_0	barometric pressure (bars)		0.98×10^6 (73.5 cm)	
11	V_0	volume of thermophone cavity (cc)		9.2	
12	S	surface of thermophone cavity (cm^2)		45.8	
13	a	area of one side of strips (cm^2)		5.08	
14	I_0	direct current in strip (amp.)		0.500	
15	I	alternating current in strip (amp.)		0.005	
16	R	resistance of strip with I_0 (ohms)	5.010		5.412
17	R_0	resistance of strip without I_0(ohms)	4.883		4.883
18	m	mass per sq. cm of strip (g)		160×10^{-6}	
19	f	frequency (cycles)		variable	
*20	l	length of strip (cm)		2.54	
21	E	open circuit voltage in transmitter (mv) at 50 cycles	77.8		84.0 †
		1000 cycles	2.81		2.10 †
		5000 cycles	0.408		0.204 †

* Used only in correction factor M.

† Assuming microphone sensitivity 2.5 mv per bar. $M=0.92$ at 50 and 1000, 1.0 at 5000~.

formula, but rather how well measurements based on a given formula could be checked. Accordingly, the formula used was that given by Wente,[3] except that the correction factor M was taken as 0.92 up to 2500 cycles, and tapered from that value to 1.0 at 5000 cycles as given by Sivian.[4] Other investigators can easily compare our data with theirs by making the necessary corrections for differences in formula.

TABLE II. *Effect of variations in the factors in the thermophone formula. Change of computed pressure due to a 10 percent increase in the constant indicated.*

Type		Hydrogen Gold			Air Gold		
Frequency		50	1000	5000	50	1000	5000
1	C_p	−9.2%	−6.6%	−3.8%	−10.4%	−8.7%	−6.3%
2	ρ_0	−9.2	−6.6	−3.8	−10.4	−8.7	−6.3
3	K_0	+0.9	+1.6	+2.9	+1.1	+2.7	+4.3
4	k	+10.9	+10.7	+10.6	+12.9	+12.8	+13.2
5	C_p'	−0.6	−4.3	−6.1	−3.6	−7.1	−7.6
—							
—							
8	α_r	<½	<½	<½	<½	<½	<½
9	T_r	−0.8	−1.4	−2.0	−3.3	−3.9	−4.1
10	P_0	0.0	+2.9	+5.8	−1.4	+0.5	+3.2
11	V_0	−10.0	−10.0	−10.0	−10.0	−10.0	−10.0
12	S	<½	<½	<½	<½	<½	<½
13	a	<½	<½	<½	<½	<½	<½
14	I_0*	+10.0	+10.0	+10.0	+10.0	+10.0	+10.0
15	I	+10.0	+10.0	+10.0	+10.0	+10.0	+10.0
16	R*	+10.0	+10.0	+10.0	+10.0	+10.0	+10.0
17	R_0*	<½	<½	<½	<½	<½	<½
18	m	−0.6	−4.3	−6.1	−3.6	−7.1	−7.6
19	f	−9.7	−10.7	−9.8	−13.6	−14.6	−14.5

* Assuming $R-R_0$ to be unchanged.

Fourteen of the factors enter the computation of M. At 1000 and 5000 cycles a change of 10 percent in the value of any of the constants affects the result by less than 1 percent for hydrogen and less than ½ percent for air. At 50 cycles the effects are about twice as great.

The computation of sound pressure in a thermophone by the Wente formula includes 20 independently determined factors. This computation together with the measurement of the open circuit voltage generated in the microphone, allows the sensitivity to be determined. These 21 factors, together with typical values for air and hydrogen are listed in Table I. In this table the factors are divided into two groups; hand-

[3] E. C. Wente, *The Thermophone*, Phys. Rev. **19**, 333 (1922).

[4] L. J. Sivian, *Absolute Calibration of Condenser Transmitters*, Bell Sys. Tech. J. **X**, 96 (1931).

book values and measured values. Ballantine[5] has pointed out that a number of the constants of the former group are not very well known. Changes in the values of these constants obviously affect the check between different methods, but for purposes of comparing calibrations by this method, different investigators can either agree on the same values for these constants, or compute the effects of different choices. The accuracy of the measured factors lies largely with the investigator.

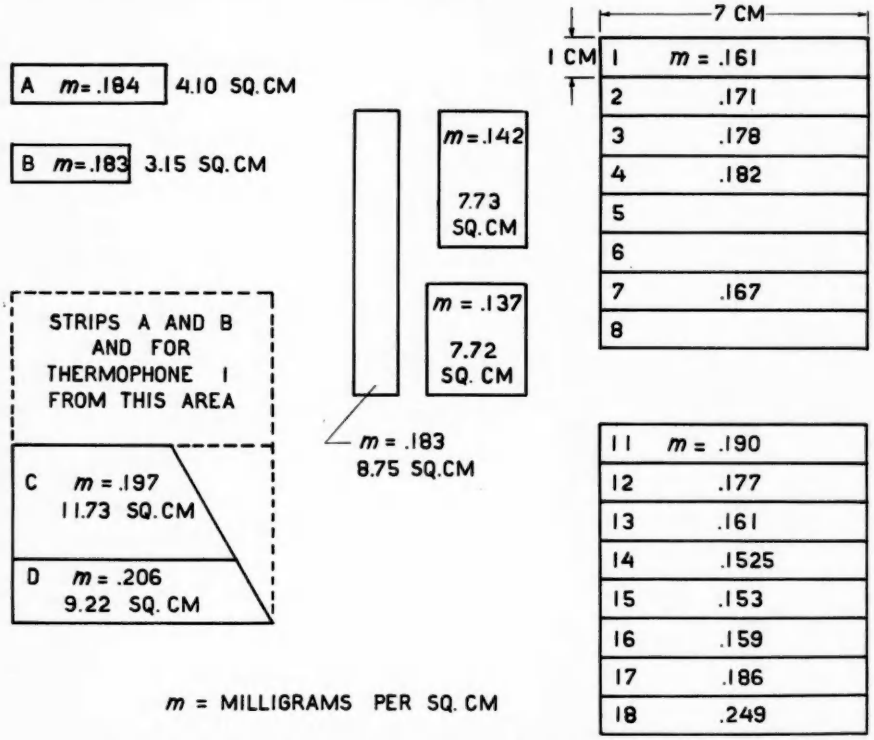


FIG. 1. *Variations in m, the mass per sq. cm in different sheets of gold leaf.*

While the 21 factors listed in Table I must be determined independently, they are not physically independent. For example, the strip thickness, which does not appear specifically, affects both the resistance and the mass per unit area of the strip, and the temperature enters in several places. The formula is quite remarkable in that nearly every factor enters at least twice, and that in practically every case the tendency is to offset the effect of a change in the value of the factor. The nature

[5] Stuart Ballantine, *Technique of Microphone Calibration*, J. Acous. Soc. Am. **III**, 319 (1932).

of the formula is such that it is next to impossible to estimate the final effect of a given uncertainty in the value of the various factors. In order to obtain some idea of the effect of such uncertainties, a computation was made for a typical set of values for three different frequencies, followed by a computation in which one of the factors was increased 10 percent. This rather laborious process was repeated for each of the 20 factors, and the results are shown in Table II. It will be observed that

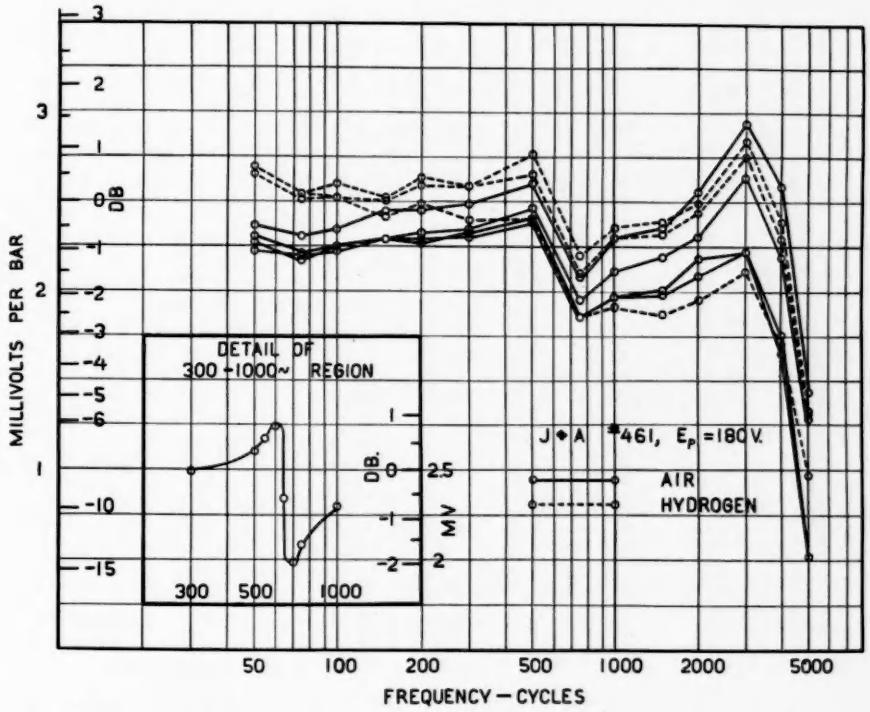


FIG. 2. *Thermophone calibrations of Jenkins and Adair Microphone, Serial No. 461. Time interval covered about one month.*

the effect of a given change depends upon the gas used in the thermophone, the frequency, and the values of the other constants. In general none of the factors seem to affect the result to a degree much above the first power, and in many cases the effect of the change is very slight.

The determination of the values of the 14 measured factors to such an accuracy that the individual effects on the result do not exceed say $\frac{1}{2}$ percent can be achieved with reasonable care with most of the factors.

The author found that the determination of the mass per unit area of the strip was perhaps the most difficult. The strips used were 1 cm x 1

in., weighed about 0.3 milligrams, and were very difficult to handle. It was found that the density varied greatly from portion to portion of a single leaf of foil as shown in Fig. 1. Strips 1 x 7 cm were cut and weighed, and the center portions of selected pairs of these strips used, but there still appears to be considerable uncertainty in this factor.

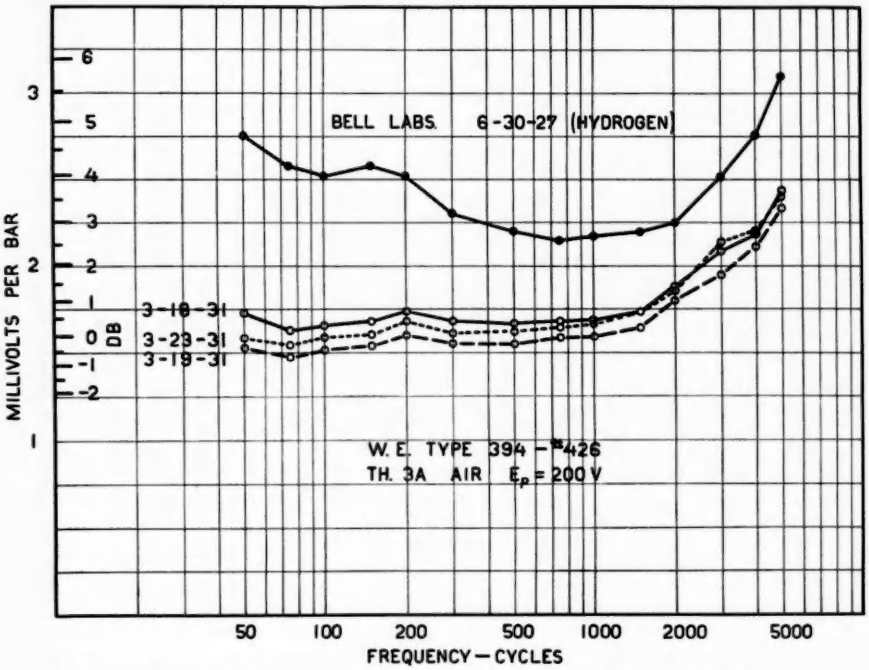


FIG. 3. *Thermophone calibrations of Western Electric Type 394, Serial No. 426 microphone showing loss of sensitivity in four year period.*

RESULTS

Fig. 2 shows seven independent calibrations which were made on Jenkins and Adair Transmitter Serial No. 461. The time interval covered was about a month, and both hydrogen and air-filled thermophones were used. It will be observed that on the average the two types of calibration agreed very well for frequencies above about 300 cycles, while below this the air calibrations averaged about 1 db less than the hydrogen. This difference might be inherent in the formula or it might be that the assumed values for the correction factor M did not apply accurately in the case of air, or for this microphone which is of somewhat different construction than the Western Electric Type 394.

The outstanding results of this series of measurements were (1) the

large change of response with frequency, particularly the abrupt drop at about 625 cycles and the falling off above 3000 cycles, and (2) the comparatively large fluctuation in sensitivity with time, particularly at the higher frequencies.

Fig. 3 shows three independent calibrations of Western Electric Type

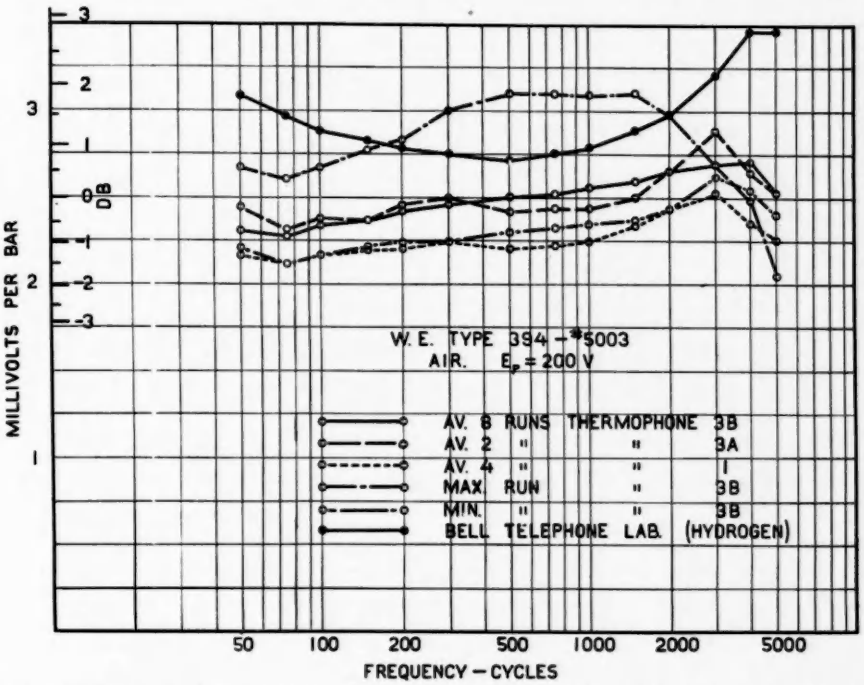


FIG. 4. *Calibration data on Western Electric Type 394, Serial No. 5003 showing variations in sensitivity and frequency-response, also check between laboratories.*

394 microphone, serial 426. These were taken with exactly the same equipment and show definite evidence of variations of frequency response and sensitivity. The upper curve shows the curve for this instrument which was furnished three years previously by the Bell Telephone Laboratories. The difference in slope at the lower frequencies might be accounted for by differences between air and hydrogen calibration, and differences in the value of M used, but these differences are slight above 1000 cycles, and there appeared to be little doubt that this particular microphone had dropped 3 or 4 db in sensitivity since the original calibration.

Fig. 4 shows a summary of the results on Western Electric Type 394 microphone serial 5003 in which we were most interested. The full line

with open circles shows the average of eight independent calibrations on the best thermophone, and represents the calibration used in working up measurements. The maximum and minimum curves included in this average are shown by the dash-dot and dot-dash-dot lines, respectively. Two of the other curves show the averages obtained with two other thermophones. As shown in Fig. 5, these measurements cover an inter-

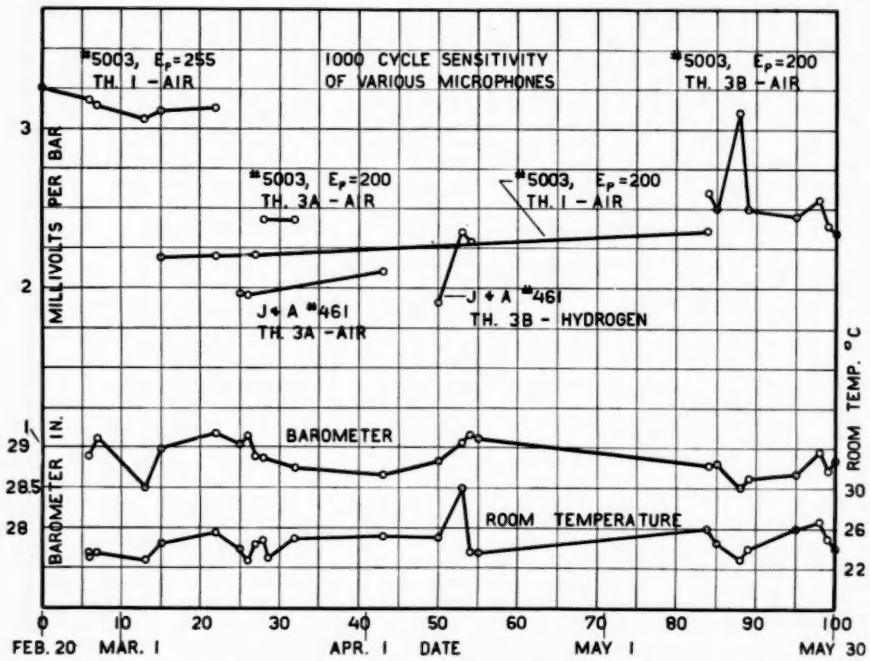


FIG. 5. *Summary of calibrations of different microphones at 1000 cycles at different times. The lines connecting the observations have no meaning.*

val of about three months, and the Bell Laboratories calibration shown in the full line with solid circles was taken at about the middle of this period. Except for the previously discussed differences at the lower frequencies, this curve lies within the range of our observations. While ordinarily the sensitivity of this microphone fluctuated over a range of about one db, the extreme range observed was about three db. There were also differences of several decibels in the frequency response curve, particularly at the higher frequencies. It is suspected that even more marked differences would have been observed if the measurements had been extended to higher frequencies.

Fig. 5 shows a summary of all the measurements taken at 1000 cycles, together with the corresponding barometer and temperature readings.

These data indicate that there is little or no tendency toward steady drifts, but that the sensitivity of all the microphones varied erratically. It also appeared that these variations could not be corrected by observations on barometric pressure or temperature. Obviously the lines between the observations have no meaning as the observations were not taken at sufficiently close intervals to follow the fluctuations.

TABLE III. *Effect of polarizing voltage on microphone sensitivity.* Western Electric Type 394 No. 5003. 400 cycles.

Polarizing voltage	*Microphone Sensitivity*	
	mv per bar	mv per bar per volt of polarizing voltage
63.5	0.705	0.0111
84.3	0.952	0.0113
106.0	1.20	0.0114
127.0	1.47	0.0116
148.5	1.74	0.0117
169.7	2.02	0.0119
190.5	2.36	0.01235
212.4	2.67	0.0126
233.5	3.06	0.0131
255.0	3.53	0.0138
276.0	4.10	0.0148

The sensitivity of a condenser microphone obviously depends upon the polarizing voltage. When sound measurements are taken near electrical machinery and other locations where it is difficult to obtain sufficient shielding from electromagnetic and similar disturbances, it is often advantageous to use a larger polarizing voltage to increase the ratio of signal to disturbance. As shown in Table III the change of sensitivity is considerably greater than the change in polarizing voltage which produces it, particularly at higher polarizing voltages. Ordinary changes of battery voltage may produce appreciable effects in some cases. Ballantine[5] has pointed out that the frequency characteristic as well as the sensitivity of the microphone is affected by change of polarizing voltage, but this effect did not appear to be very great with the range of voltages and frequencies used on the particular microphone tested.

CONCLUSIONS

These data show that the microphones exhibited variations in sensitivity with time of the order of several decibels, and thus precluded an accurate check between laboratories or between different thermophones

or thermophone atmospheres at this laboratory by the methods used. The absolute value of the Bell Laboratories calibration fell within the range of our day to day variation, and in the neighborhood of 200 to 2000 cycles agreed with our average value to within a decibel. In view of these facts it did not appear worth while to continue the investigation further even though a more accurate calibration would have been desirable.

This investigation was not entirely satisfactory for determining the absolute calibration at the time the measurements on the gears[2] were taken, although the preliminary calibration in the neighborhood of 1000 cycles which was made during the course of the gear noise measurements agreed with the final value to within 0.2 db! As stated in the previous paper it was found that readings taken on a given gear unit on different days varied over a range of approximately a decibel, and that several independent sets of measurements were made on each machine to average out this effect. It is believed that this also tended to average out variations in microphone sensitivity. It is not believed that variations in frequency-response were serious in this case although they might be in others.

It appears that in many applications of soundmeters these fluctuations in microphone sensitivity are sufficiently large to be very important, and means should be taken by the investigator to take account of this variable. In certain types of work it is planned to take a thermophone into the field so that frequent calibrations can be made at a few frequencies at least. For this purpose the air-filled thermophone has obvious advantages, even though it is less sensitive than the hydrogen, develops much higher temperature in the strip for given currents which tends to shorten the life, and is not suitable for the higher frequencies. It also appears that investigations into the stability of microphones should be of great interest to the users of sound measuring equipment.

May 2, 1932

SOME IMPROVEMENTS ON THE OPERATION OF THE BELLS OF A CARILLON

By G. M. GIANNINI
Curtis Institute of Music

ABSTRACT

The unsatisfactory performance of the bells of a carillon from a musical standpoint is discussed and an improvement is suggested. This is the use of a damping device to permit greater musical possibilities in the playing of carillons. The operation of the electrical apparatus which performs the damping is described.

In most musical instruments the main overtones are in harmonic relation to each other and the ratio of their amplitude is fairly constant during the time the emission of sound lasts. This condition, however, does not exist in a bell, or in other words the overtones of the bell are not in harmonic series. The vibrations vary continuously in amplitude at various points of its surface and various overtones are successively predominating during the period of radiation of the sound.

One of these overtones is a minor or major third, which is musically undesirable and makes the average listeners think the bells are out of tune. With a minor third, which is so commonly predominating even in the best bells available today, it is still possible to obtain musical combinations of some artistic value, if melodies are chosen in which tunes are played slowly, thus avoiding dissonant combinational tones.

Another severe limitation existing in the carillon is due to the fact that the sound of the large bells lasts longer than the sound of the small ones, which often completely deceives the true character of the music. This limitation can be overcome with the use of dampers. The utility of a device able to control the volume of sound does not have to be emphasized since a damping effect is used on most all musical instruments. The player should be able to damp one or more bells of the carillon as the occasion demands.

Bell dampers have been designed in the past, consisting of a piece of soft material sometimes connected with the clapper. This device acted from the bell clapper and pressed toward the sound bow of the bell. Such a type of damper never became popular, possibly because it acted every time the bell was struck, thus changing the true character of the bell tone. Moreover the physical effort necessary to operate such a device from the keyboard was too much for an average man who already needed all his strength to perform the normal playing of a carillon.

A correct type of damper is one that can be easily operated from the

clavier without muscular strain, and act only at the will of the player, with various speeds, on one or more bells. A device satisfying these requirements has been successfully tested on bells of the Bok Tower, Mountain Lake, Florida.

The first problem was to find the best location on the surface of the bell to damp it effectively. This region has been found to be the sound bow, where the resulting amplitudes of all the vibrations is at a maximum. Damping contacts of a few square inches of leather were tried.

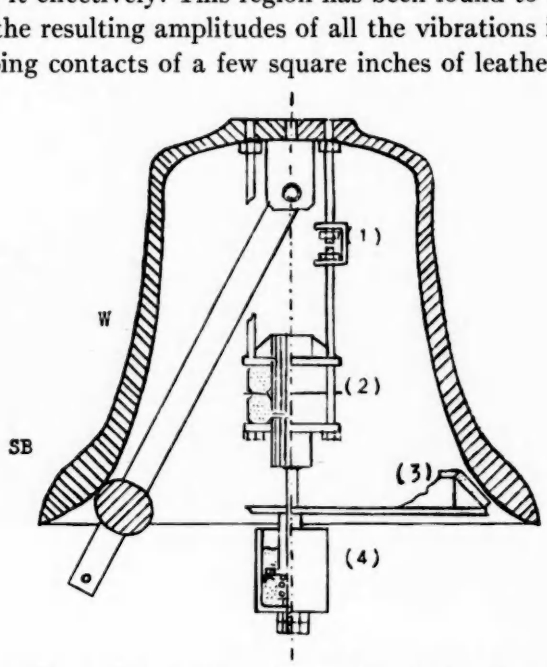


FIG. 1. *Bell damper; W, waist; SB, sound-bow. (1) acoustic suspension; (2) magnet; (3) damper; (4) dash-pot.*

first on a single and later on two diametrical points on the sound bow. The action produced some unpleasant high tones during the period of damping because of the high pressure applied on the small surface. The next test then was to increase the surface of damping contact and use a softer material: a circular ribbon of felt, mounted on a light wood and steel frame gave satisfactory results. This damping ring is not a complete circle, room being left for the swing of the clapper. The width of the felt ribbon is 1/20 of the height of the bell, this ratio being the most effective of the several which were tried.

The damper is connected with the armature of an electromagnet, which is hung inside the bell. The magnet used was of the standard Westinghouse line. The magnet structure was first energized by 60 cycles alternate current source. The operation was extremely noisy since even the slightest buzzing was transmitted through the steel frame

of the carillon, becoming quite objectionable. Different sound insulating materials were employed during the tests but with insufficient results.

Successful operation was obtained with the use of d.c. power, which not only avoided any kind of buzzing, but made possible the insertion of a thin rubber spacer between the magnet core and the armature. The power was furnished by a battery, which was kept continuously charged by a tungar rectifier. Because of the instantaneous heavy current loads, batteries were chosen, as the best source of d.c. power.

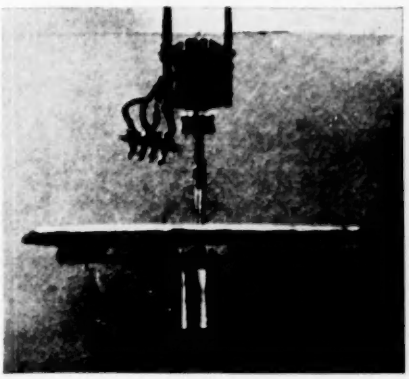


FIG. 2.

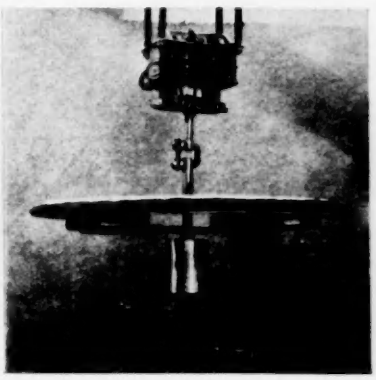


FIG. 3.

The final arrangement includes a dash-pot inserted between the magnet and the damper which smooths the damping action on the bell. The dash-pot has an adjustment which permits of varying the damping speed. The oil used in the dash-pot is of the electric-transformer type, which does not change too much in viscosity with change of temperature.

The damper equipment needs little attention, once the dash-pots are adjusted, since it is well shielded from the weather, and, of primary importance, does not impair any of the desirable properties of the carillon. Fig. 1 shows the magnet, the damper and the dash-pot inside the bell, while Figs. 2 and 3 show the practical assembling of the unit, with the magnet housing removed.

Not much energy is needed to stop a playing bell. In the middle octave it was found that if the damper was held against the sound-bow with a pressure equal to the weight of the clapper, good results were obtained. The ratio of the weight of the bell to the weight of the clapper is not strictly proportional over a range of several octaves: with the heavier bells the ratio is about 1/100 while in the lighter ones it becomes 1/10. The energy communicated by the clappers to the bells, however, does not vary in the same way, because with all the clappers moving

through the same length of stroke, the light ones have a higher speed than the heavy ones. Things are evidently arranged in this way by the bell founders in the effort to compensate the fast decay of the sound of the small bells and to obtain as far as possible the same amount of sound from all the bells. Because of their fast decay, which permits a rapid playing, no dampers were found necessary on the bells higher than the second *C* above the middle *C*.

Dampers are operated through relays by switches located on the clavier. These switches can be connected with push buttons or pedals following the personal taste of the carilloneur. He can decide also about the kind of operation which he prefers, since the circuits are easily arranged in order to energize the dampers individually, by octaves or all simultaneously. Each damper magnet has two separate coils, which can be energized in series, in parallel, or individually, thus allowing different damping speeds.

When all the bells are damped at the same time an interlock system prevents overloading of the battery, energizing first the dampers of the bells that require a longer time to become silent and then the others, so that the practical level of inaudibility is reached by all the bells simultaneously.

An automatic action allows any particular bell to be damped soon after it is struck. Such an action is able to keep all the bells permanently damped and release only the bells which are played for a given time after the stroke. This action, which is analogous to that of the dampers in a piano, prevents interaction between different bells. By actual measurements it has been found that if a middle size bell is struck a number of times until the level of the sound reaches about 100 db at 200 feet from the belfry, and then the bell is suddenly damped, all the other bells bells emit a sound which can reach 50 db. This sound, due to the above-mentioned interaction, is quite objectionable when the noise level is low. Since in old carillons mounted on wooden frames, this intruding sound is not present, it seems that this interaction is due to transmission of sound through the metallic frame and that the bells are not acoustically insulated from their supports. In fact the pieces of leather commonly used under the bolts which hang the bells are so much compressed that they practically have lost all their sound absorbing power.

In conclusion, I wish to express my appreciation to Maestro Rosario Scalero of the Curtis Institute of Music of Philadelphia, and to Mr. F. C. Godfrey of John Taylor and, Bellfounders, for their musical guidance and technical suggestions in working out problems encountered.

A NEW ANALOGY BETWEEN MECHANICAL AND ELECTRICAL SYSTEMS

By F. A. FIRESTONE
University of Michigan

ABSTRACT

By considering each mass in a linear mechanical system as having two terminals, one fixed in the mass and one fixed to the frame of reference, every linear mechanical system is reduced to a multiplicity of closed mechanical circuits to which force and velocity relations similar to Kirchhoff's laws, may be applied. The conventional mechanical-electrical analogy is derived from the similarity of the equations $v=f/z$ and $I=E/Z$. It is incomplete in the following respects which lead to difficulty in its application.

I. There is a lack of analogy in the use of the words "through" and "across" which indicates a fundamental difference in the nature of the analogous quantities, for instance, force through and e.m.f. across.

II. Mechanical elements in series must be represented by electrical elements in parallel, and *vice versa*.

III. Mechanical impedances in series must be combined as the reciprocal of the sum of the reciprocals while electrical impedances in series are additive.

IV. There is an incompleteness in the mechanical analogues of Kirchhoff's laws.

The new analogy is derived from the similarity of the following equations: $v=f\bar{z}$ and $E=IZ$ where $\bar{z}$ is the reciprocal of the mechanical impedance as usually defined. This new analogy is complete in all of the above-mentioned respects in which the old analogy failed. It leads to analogous relations of a simple sort and permits an equivalent electrical circuit to be drawn in an easy intuitive manner.

INTRODUCTION

If a group of physical concepts or quantities are related to each other in a certain manner, as by equations of a certain form, and another group of concepts or quantities are interrelated in a similar manner, then an analogy may be said to exist between the concepts of the one group and those of the other group. The essential element of an analogy is a similarity between the relations within one group and the relations within the other group. Two groups of concepts may be analogous in some respects and not analogous in other respects.

An analogy is valuable to the extent that it permits a knowledge of one field to be applied in another field. Since much more is known of the characteristics of electrical circuits than of certain kinds of mechanical systems, it is often valuable to discuss a mechanical system in terms of its electrical analogue. It will be shown that the conventional mechanical-electrical analogy is incomplete in certain important particulars which make it difficult to apply in practice. A new kind of mechanical-electrical analogy is set forth below which is more complete than the old and permits an equivalent electrical circuit to be drawn in

a much more straightforward and common-sense manner, not requiring such careful reasoning at each step.

We have all learned at school that inductance in an electrical circuit plays a part similar to mass in a mechanical system.[1] In substantiation of this viewpoint it is mentioned that the energy stored in the magnetic field of the inductance is $\frac{1}{2}\ LI^2$ and the kinetic energy of a mass is $\frac{1}{2}\ mv^2$; that the inductance tends to prevent a change of current by generating a back e.m.f. of magnitude LdI/dT just as a mass tends to prevent a change of velocity by producing a reacting force of magnitude mdv/dt. But one might say with equal truth that capacity plays the rôle of electrical mass because the energy stored in the electrostatic field of a condenser, $\frac{1}{2}\ CE^2$, corresponds to the kinetic energy of a mass, $\frac{1}{2}\ mv^2$; and also that the condenser tends to prevent a change of e.m.f. by absorbing a current of magnitude CdE/dt just as a mass tends to prevent a change of velocity by producing a reacting force of magnitude mdv/dt.

Similarly we have been told that capacity in an electrical circuit plays a part similar to the compliance c of a spring in a mechanical system. And it is pointed out that the energy stored in the electrostatic field of the condenser is $\frac{1}{2}\ CE^2$ corresponding to the energy $\frac{1}{2}\ cf^2$ stored in a spring by a force f; that a condenser will hold a charge, CE, proportional to the e.m.f. while a spring undergoes a displacement, cf, proportional to the applied force. But it may be said with equal reason that inductance plays the part of electrical stiffness because the energy stored in the magnetic field of the inductance, $\frac{1}{2}\ LI^2$, corresponds to the energy stored by a spring, $\frac{1}{2}\ cf^2$; and also that an inductance will store a voltage impulse ($\int Edt$) of magnitude LI proportional to the current while a spring undergoes a displacement, cf, proportional to the applied force.[2]

It is, therefore, evident that a new analogy is possible in which force is identified with current rather than with e.m.f. as in the old analogy. In what follows, it will be shown that the new analogy is more complete and more useful than the old.

THE MECHANICAL ELEMENTS

We will confine our attention to the consideration of those mechanical systems wherein all the forces and the resulting velocities are essentially

[1] For instance, Starling, *Electricity and Magnetism*, p. 307.

[2] In Liven's, *Theory of Electricity*, p. 416, it is said "It must, however, be particularly emphasized that we have no definite proof that this (magnetic) energy is kinetic, it is merely a matter of convenient choice so to regard it."

in one line. Many vibrating systems, including torsional systems, can be very simply reduced to such a linear problem. The elements of a mechanical system are assumed to be of three kinds, springs, resistances and masses, although practically it may be difficult to realize any one of them in its pure form. We may assume that all of the elements lie in one horizontal line and that the positive direction along this line is from left to right. In order to understand the analogy in detail, it is necessary to look carefully at the nature of each kind of mechanical element and to be sure that we have a clear conception of the meaning of the quantities which may be used in describing its state.

A spring has two terminals to which force may be applied, and since the force is the same in all parts of the spring we will speak of the "force through the spring." The force through the spring may be assumed to be positive when the spring is under tension, and the fact that a spring is under tension may be indicated by an arrow in the positive

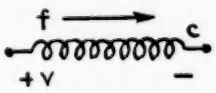


FIG. 1. *Spring.*

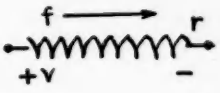


FIG. 2. *Resistance.*

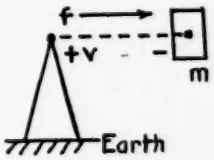


FIG. 3. *Mass.*

direction (to the right) as shown in Fig. 1. This is merely a convention which must be remembered. There will always be a positive force through a spring when it is longer than its unstrained length. If the left terminal of a spring through which there is a positive force (tension) is attached to some object, then the spring lies on the positive side of the object and the object will be subjected to a force in the positive direction. If the right terminal of the spring is attached to the object, then the spring lies on the negative side of the object, and the positive force through the spring will pull the object in the negative direction.

The relative velocity of the terminals will be called the "velocity difference across" the spring or simply the "velocity across." The velocity difference is counted positive at a given terminal if in imagining that terminal as fixed, the other terminal is moving in the positive direction. This may be indicated by + and − signs as in Fig. 1. Thus if we stand at the left end of the spring and imagine it as fixed, the plus sign by our side will indicate that the other terminal is moving in the positive direction (to the right in the figure) or away from us. Similarly, the negative at the right end indicates that if the right end is fixed, the left end will be moving in the negative direction (to the left). Conse-

quently, a plus sign at the left terminal of a spring indicates that the spring is growing longer, and *vice versa*. (In a torsional spring there is no essential distinction between the two possible directions of twist analogous to lengthening and shortening in a linear spring. One may merely assume arbitrarily that a counterclockwise velocity of the nearer terminal relative to the farther is a positive angular velocity difference across the spring; and likewise that a counterclockwise torque on the nearer terminal sends a positive torque through the spring.)

A mechanical resistance may be visualized as two massless concentric tubes with a layer of viscous oil between. It also has two terminals. With the same conventions as in the previous paragraph and as indicated in Fig. 2, a resistance will have a positive force through it (tension) when the velocity difference across it is positive at the left terminal (resistance growing longer).

It is not obvious that a mass has two terminals. One cannot apply a force to a spring or to a resistance without grasping it at two points, but force can apparently be applied to a mass by contacting it at one point only. But a force is itself a two-terminal element since action and reaction are equal and opposite; it is impossible to push or pull without standing on something. Thus in order that a force may act on a mass, the force must react against another mass, which may be the earth. The acceleration produced on a mass by a given force is the same whether that force reacts against a small mass or against the earth. Since we measure the velocity of the mass relative to the earth,[3] and since any impressed force must in effect react against the earth, it is helpful to assume that every mass has two terminals, one of which is some arbitrary point of the mass, the other being a point near the mass, which is fixed to the earth, as shown in Fig. 3. Here again the velocity difference across the mass (relative velocity) of its terminals is considered positive at the left terminal when the right terminal is moving to the right. (While in ordinary language we speak of the velocity *of* the mass instead of the velocity difference *across* the mass, this is similar to the case of any two terminal electrical element, one side of which is grounded, wherein the potential *of* the high side is the same as the potential difference *across* the element.) If we impress on a mass an external positive force f' as shown in Fig. 4, the mass will receive an acceleration which will be positive at the left terminal and which will call forth a reactive tensile force between the terminals of the mass

[3] Any unaccelerated frame of reference would do equally well.

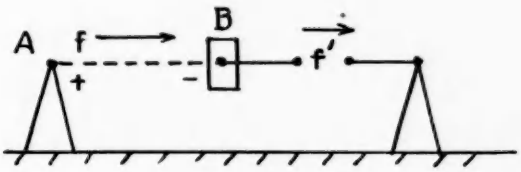
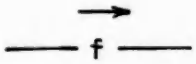
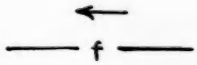


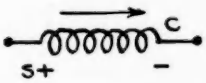
FIG. 4.



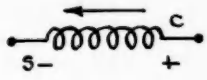
A tensile force tending to pull these terminals together.



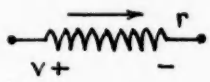
A compressive force tending to push these terminals apart.



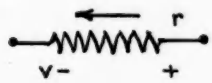
A spring in tension. The displacement differences must therefore be positive at the left terminal, meaning that as seen from the left terminal the right terminal must have moved in the positive direction. Spring lengthened.



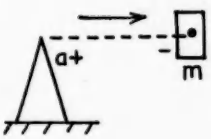
A spring in compression. The displacement difference is therefore negative at the left terminal. Spring shortened.



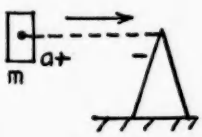
Resistance in tension. The velocity difference must therefore have the sign shown, resistance growing longer.



Resistance in compression, growing shorter.



Mass with acceleration difference of signs shown must be in tension. The right terminal is accelerating toward the right.



This mass with acceleration difference as shown must be in tension. The left terminal is accelerating to the left.

FIG. 5.

A and B, of magnitude $f=ma$. Thus the tensile force f' has produced a tensile force through the mass to the earth.[4] It is as if the earth[4] did not like to see masses accelerated and acted on them with a restraining force. Assuming as before that a tensile force through an element is positive, that is, the element is trying to pull its terminals together, there is a positive force through a mass when there is an acceleration across it which is positive at the left terminal. Similar conventions can be worked out for a moment of inertia in a torsional system.

The above conventions are summarized and exemplified in Fig. 5.

Mechanical elements will be assumed to be connected mechanically in series when they are joined end to end as shown in Fig. 6. In such an arrangement is it obvious that the force through all the elements is equal; if one element is in tension they are all in tension. Likewise the velocity difference across a series of elements is the algebraic sum of the velocity

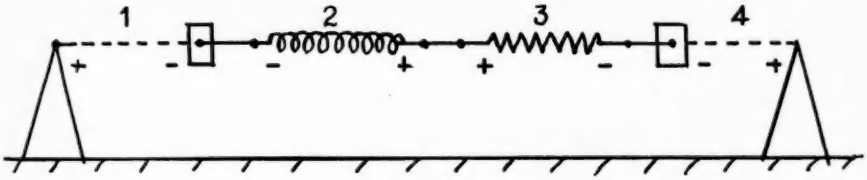


FIG. 6.

differences across each element. When a number of mechanical elements are connected to two common junction points, as shown in Fig. 7, they may be said to be connected in parallel. The connecting bar on the right is assumed to remain vertical. It is obvious that in such an arrangement the velocity difference across all of the elements is the same. Furthermore the total force through the combination is the algebraic sum of the separate forces through the elements.

Springs and resistances may be connected either in series or in parallel but a number of masses can be connected in parallel only, as one terminal of each mass is connected to the earth. Two masses only may be connected in series through other elements as shown in Fig. 6, but if they were connected directly together, they would in effect be in parallel. Of course, springs and resistances may be connected either in series or in parallel with masses.

We may state two additional relations which hold for any linear mechanical system, the force law and the velocity law. According to the force law, the algebraic sum of all the forces acting on any junction

[4] Or other frame of reference.

of mechanical elements is zero. For instance if we consider the central junction in Fig. 7 and assume that a force whose arrow is away from the junction should be counted positive, then

$$f - f_1 - f_2 - f_3 - f_4 = 0.$$

The velocity law states that the algebraic sum of the velocity differences around any closed mechanical circuit is zero. It is necessary to take as the sign of each velocity difference the first sign which is seen on approaching the element in going around the circuit. Thus in Fig. 6 if the signs of the velocities are assumed as shown,

$$v_1 - v_2 + v_3 - v_4 = 0.$$

Also in Fig. 10 the velocity law applied around the central circuit would give $v_3+v_4+v_5-v_6=0$.

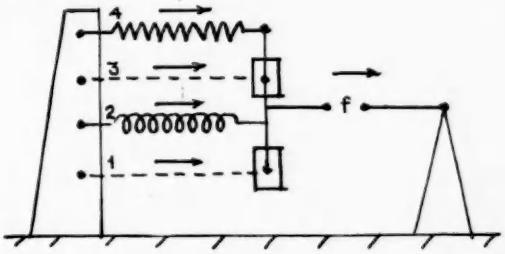


FIG. 7.

We follow the usual convention in defining mechanical impedance as the complex quotient of the force through and the velocity difference across an element or combination of elements.

$$z = f/v.$$

(This definition was originally chosen with the conventional analogy in mind.) In a series of mechanical elements having individual impedances of z_1, z_2, z_3, etc., the impedance of the combination would be

$$z = \frac{f}{v_1 + v_2 + v_3 + +} = \frac{1}{\frac{v_1}{f} + \frac{v_2}{f} + \frac{v_3}{f} + +} = \frac{1}{\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3} + +}.$$

Thus mechanical impedances in series add as the reciprocal of the sum of the reciprocals as do electrical impedances in parallel. Similarly if we have a number of mechanical impedances in parallel, the impedance of the combination is

$$z = (f_1 + f_2 + f_3 + +)/v = z_1 + z_2 + z_3 + + .$$

So mechanical impedances in parallel are additive like electrical impedances in series.

In the above description of the mechanical elements and the mechanical circuit no detailed assumption has been made as to the nature of any analogy with the electrical circuit which may be noted later, although some of the conventions were suggested by analogy. By considering each mass as having one terminal fixed to the frame of reference, every mechanical system is reduced to a multiplicity of closed mechanical circuits, thereby preparing the way for the application of electrical analogies.

THE CONVENTIONAL ANALOGY

The conventional mechanical-electrical analogy may be derived from the fact that for most mechanical systems, an electrical system can be invented of such a sort that the differential equations of motion in the two systems, as expressed in terms of displacement and charge, respectively, will be of the same form. If impedances are defined as in the following vectorial equations

$$v = f/z \text{ (mechanical)} \quad I = E/Z \text{ (electrical)}$$

the form of the impedances as derived from the differential equations will be similar and will justify the following conventional analogy.

THE CONVENTIONAL ANALOGY

Mechanical	*Electrical*
Force through $=f$ (dynes)	e.m.f. across $=E$ (volts)
Velocity across $=v$ (cm/sec.)	Current through $=I$ (amperes)
Displacement across $=s$ (cm)	Charge through $=Q$ (coulombs)
Impulse through $=p$ (dyne sec.)	Voltage impulse $=\int E dt$ (volt sec.)
Impedance $=z$ (ohms)*	Impedance $=Z$ (ohms)
Resistance $=r$ (ohms)	Resistance $=R$ (ohms)
Reactance $=x$ (ohms)	Reactance $=X$ (ohms)
Mass $=m$ (grams)	Inductance $=L$ (henries)
Compliance $=c$ (cm/dyne)	Capacity $=C$ (farads)
Power $=f\,v$ (ergs/sec.)	Power $=EI$ (watts)
z of resistor $=r$ (ohms)	Z of resistor $=R$ (ohms)
z of mass $=i\omega m$ (ohms)	Z of inductance $=i\omega L$ (ohms)
z of spring $=-i/\omega c$ (ohms)	Z of condenser $=-i/\omega C$ (ohms)
Impedances in series $z = \dfrac{1}{1/z_1 + 1/z_2 + 1/z_3 + }$	Impedance in series $Z = Z_1 + Z_2 + Z_3 +$
Impedances in parallel $z = z_1 + z_2 + z_3 +$	Impedances in parallel $Z = \dfrac{1}{1/Z_1 + 1/Z_2 + 1/Z_3 + }$
Force and velocity laws	Kirchhoff's laws
Sum of velocity differences around closed circuit is zero	Sum of currents to a junction is zero
Sum of forces to a junction is zero	Sum of e.m.f.'s around a mesh is zero

* Mechanical ohm = dyne/kine.

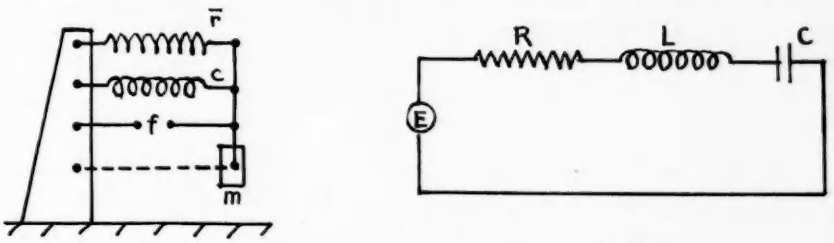
Thus in the familiar problem of the forced vibration of an elastically bound mass with friction, and its analogous circuit, shown in Fig. 8, the differential equations are

$$\frac{md^2s}{dt^2} + \frac{rds}{dt} + \frac{s}{c} = fe^{i\omega t} \qquad \frac{Ld^2Q}{dt^2} + \frac{RdQ}{dt} + \frac{Q}{C} = Ee^{i\omega t}.$$

The steady state solutions of these in vector forms are

$$v = \frac{f}{r + i\left(\omega m - \frac{1}{\omega c}\right)} = \frac{f}{z} \qquad I = \frac{E}{R + i\left(\omega L - \frac{1}{\omega C}\right)} = \frac{E}{Z}.$$

Therefore the velocity across the mass due to the impressed force is the same as the current through the inductance due to the e.m.f. E.



Mechanical system *Conventional analogue*

FIG. 8.

However, while the mechanical elements are connected in parallel, the analogous electrical elements must be connected in series. This must always be the case with this analogy since mechanical elements in parallel have a common velocity difference across them and electrical elements in series have a common current through them. In general, mechanical elements in series are represented by electrical elements in parallel, and *vice versa*, so that in a complicated mechanical system the analogous electrical elements must be placed in a manner quite contrary to intuition. A mass, which always has one terminal on the earth, will usually be represented by an inductance in the high side of the line, and a spring which is in series with the mechanical circuit and must transmit all the force from one part of the mechanical system to another will be shown as a condenser across the line with one side grounded, while a spring which connects part of the mechanical system to the earth will be represented by a condenser in series with the high wire. This is shown in Fig. 12.

The difficulty is also indicated in the use of the words "through" and "across" in the table of analogous quantities above. Force has a "through" character like current; the force through each of a series of mechanical elements is the same, just as the current through each of a series of electrical elements is the same. Also, velocity difference has an "across" character like potential difference; the velocity difference across each of a number of mechanical elements in parallel is equal, just as the potential differences across electrical elements in parallel are equal. It is in ignoring these fundamental characteristics of the analogous quantities and placing "force through" analogous to "e.m.f. across" that the conventional analogy becomes unnecessarily difficult of application.

Furthermore it is noted in the table that the laws for the addition of impedances are not analogous, and also that there is a lack of accurate correspondence between Kirchhoff's laws and their mechanical analogues.

THE NEW ANALOGY

The analogy here proposed may be derived from the similarity of the equations

$$v = f\bar{z} \quad E = IZ$$

where $\bar{z}$ is the bar impedance, the reciprocal of the mechanical impedance. The real part of $\bar{z}$ is the bar resistance $\bar{r}$, and its imaginary part, the bar reactance $\bar{x}$. Since the bar impedance of a spring of compliance c is $i\omega c$ and the impedance of an inductance L is $i\omega L$, the impedance of an inductance $L=c$ is at all frequencies equal to the bar impedance of the spring. Similarly the bar impedance of a mass m is $-i/\omega m$ and the impedance of a condenser is $-i/\omega c$ so that the impedance of a capacity $C=m$ will at all frequencies be equal to the bar impedance of the mass. The bar impedance of a mechanical resistance r ohms is $1/r$ and is therefore equal to the impedance of an electrical resistance $R=1/r$ ohms. In this way the following analogy is established.

THE NEW ANALOGY

Mechanical	*Electrical*
Force through $=f$ (dynes)	Current through $=I$ (amperes)
Velocity across $=v$ (cm/sec.)	e.m.f. across $=E$ (volts)
Displacement across $=s$ (cm)	Voltage impulse across $=\int E dt$ (volt sec.)
Impulse through $=p$ (dyne sec.)	Charge through $=Q$ (coulombs)
Bar impedance $=\bar{z}$ (ohms)	Impedance $=Z$ (ohms)

* Bar impedance = velocity across/force through.

* ohms = bar ohms = kines/dyne.

THE NEW ANALOGY (*Cont'd*)

Mechanical	*Electrical*
[b]Bar resistance $=\bar{r}$ (ohms)	Resistance $=R$ (ohms)
[c]Bar reactance $=\bar{x}$ (ohms)	Reactance $=X$ (ohms)
Mass $=m$ (grams)	Capacity $=C$ (farads)
Compliance $=c$ (cm/dyne)	Inductance $=L$ (henries)
Power $=fv$ (ergs/sec.)	Power $=IE$ (watts)
$\bar{z}$ of resistor $=\bar{r}$ (ohms)	Z of resistor $=R$ (ohms)
$\bar{z}$ of mass $=-i/\omega m$ (ohms)	Z of condenser $=-i/\omega C$ (ohms)
$\bar{z}$ of spring $=i\omega c$ (ohms)	Z of inductance $=i\omega L$ (ohms)
Bar impedances in series	Impedances in series
$\bar{z} = \bar{z}_1 + \bar{z}_2 + \bar{z}_3 +$	$Z = Z_1 + Z_2 + Z_3 +$
Bar impedances in parallel	Impedances in parallel
$\bar{z} = \dfrac{1}{1/\bar{z}_1 + 1/\bar{z}_2 + 1/\bar{z}_3 +}$	$Z = \dfrac{1}{1/Z_1 + 1/Z_2 + 1/Z_3 +}$
Force and velocity laws	Kirchhoff's laws
Sum of forces to a junction is zero	Sum of currents to a junction is zero
Sum of velocity differences around closed circuit is zero	Sum of e.m.f.'s around a mesh is zero

[b] Bar resistance = real part of bar impedance.
[c] Bar reactance = imaginary part of bar impedance.

It has seemed advisable to introduce a new term, bar impedance, which is equal to *velocity across/force through*. It is natural that the old analogists, having arrived on the ground first, should have chosen to define impedance as *force/velocity* since that fitted in with the other assumptions they had made. But in the author's opinion, all of their assumptions were unwise and led to the left-handed result that while electrical impedances in series are additive, mechanical impedances in series must be added as the reciprocal of the sum of the reciprocals as was shown above. It would have been better if this new analogy had been thought of first, for in that case the quantity which we have been forced to call "bar impedance" would have been called "impedance" and would have been subject to the same laws of addition as are found in the electrical circuit. It is now too late to change suddenly the unfortunate definition of impedance which has been so much used in the past, so it is recommended that the term "bar impedance" be used. Then if the new analogy should prove popular, the time may come when the old definition of impedance will have fallen into disuse, at which time the "bar impedance" may be shortened to "impedance" with the new definition.[5]

[5] Of course, bar impedance is the same as mechanical admittance as usually defined, but to use the latter term would spoil the analogy and cause confusion.

This nomenclature therefore prepares the way for an evolutionary change of definition. In the meantime $\bar{z}$ will indicate that this quantity is z below the line, that is, the reciprocal of z as ordinarily defined. It should be noted, however, that $\bar{r}$ is the real part of $\bar{z}$ and is equal to r/z^2, being the reciprocal of r only when $z=r$, a pure resistance; similarly for $\bar{x}$.

The unit in which bar impedance is measured is the bar ohm ($\overline{\text{ohm}}$) which is one kine per dyne. In time the bar might be dropped thereby constituting a change of definition.

In a series mechanical system, the bar impedance of the combination is the sum of the bar impedances of the elements.

$$\bar{z} = \frac{v_1 + v_2 + v_3 + \cdots}{f} = \bar{z}_1 + \bar{z}_2 + \bar{z}_3 + \cdots .$$

Similarly for a parallel mechanical system

$$\bar{z} = \frac{v}{f_1 + f_2 + f_3 + \cdots} = \frac{1}{1/\bar{z}_1 + 1/\bar{z}_2 + 1/\bar{z}_3 + \cdots} .$$

Thus the bar impedance follows the same laws of addition as electrical impedance.

The new electrical analogue for displacement is the voltage impulse,[6] ($\int Edt$) this being the electrical quantity which is measured by a ballistic galvanometer. It was given this name by the old analogists because to them it was analogous to mechanical impulse. The lack of correspondence at this point in the new analogy is again a mere matter of definition. In any problem where a simple harmonic displacement amplitude is specified, it is convenient in the new analogy to convert it to velocity by multiplying by $i\omega$.

In both analogies, mechanical power and electrical power are analogous.

It will be noted that in this analogy the words "through" and "across" are analogously used. In the old analogy it was appealing to have force and electromotive force be analogues because of the similar sound of the words which, when loosely used, appear to have a similarity of meaning; but when one analyzes the case more closely and finds that force *through* is analogous to electromotive force *across*, some of the attractiveness is lost. One may also favor the old analogy if he thinks of e.m.f. as cause and current as effect, of force as cause and velocity as

[6] Bennet and Crothers, *Introductory Electrodynamics for Engineers*, p. 470.

effect. However, in a series electrical circuit it is more convenient to consider the current through an element as the cause and the e.m.f. across the element as the effect, and in a series mechanical system, of force through an element as cause and velocity across the element as effect, thereby in that case favoring the new analogy.

With this analogy also, the differential equations of motion in the analogous systems will be of the same form if written in appropriate variables. For instance, consider the problem in Fig. 9 where a velocity

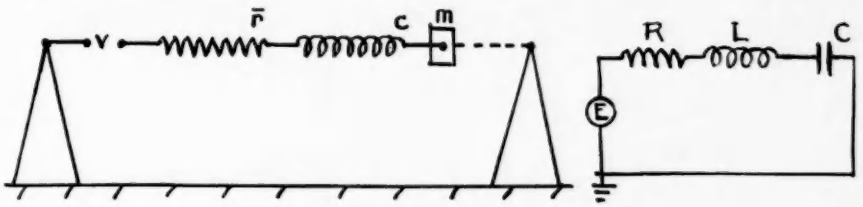


FIG. 9.

$ve^{i\omega t}$ is impressed in series with a resistance, spring, and mass and we wish to find the resulting force through the system. Applying the velocity law around the mechanical circuit and using the impulse $p = \int f dt$ as dependent variable we get the following differential equations:

$$c\frac{d^2p}{dt^2} + \bar{r}\frac{dp}{dt} + \frac{p}{m} = ve^{i\omega t}; \quad \frac{Ld^2Q}{dt^2} + \frac{RdQ}{dt} + \frac{Q}{C} = Ee^{i\omega t}.$$

Since $f = dp/dt$ and $I = dQ/dt$ the steady state solutions of these equations in vectorial form are

$$f = \frac{v}{\bar{r} + i(\omega c - 1/\omega m)} = \frac{v}{\bar{z}}; \quad I = \frac{E}{R + i(\omega L - 1/\omega c)} = \frac{E}{Z}.$$

The velocity across any element can be found by multiplying the above value of f by the bar impedance of the element.

In the above differential equations and their solutions the analogy as set forth in the above table is obvious. In the series mechanical circuit shown, the impulse through each element is the same while the displacements across them are different, consequently it is more convenient to write the differential equation with impulse as dependent variable, than displacement. We may therefore say that if the founders of vibration theory had happened to consider the forced vibration problem of the series mechanical system with impressed velocity, instead of the parallel system with impressed force, the analogy here presented is the

one which they would have noted as being the most obvious and appealing.

EXAMPLES

The above differential equations have been presented for the benefit of those who feel that an analogy should come from a similarity of differential equations. However, when one wishes to find the steady state e.m.f.'s and currents in a given electrical circuit he does not write down any differential equations. He finds the impedances of the elements at the given frequency by well-known formulae, applies Kirchhoff's first law to the currents entering the junction points, applies Kirchhoff's second law to the voltages around each mesh, and solves the simultaneous equations thus derived for the unknown currents and voltages. It is, therefore, desirable that one should not have to write the differential equations for either the mechanical or the electrical system in finding the electrical analogue of a given mechanical system.

In the new analogy, the equivalent electrical circuit is drawn to resemble the original mechanical system, remembering that one terminal of each mass is connected to earth. Each spring is represented by an inductance $L=c$; each mass is represented by a capacity $C=m$ one side of which is connected to ground; each mechanical resistance of bar resistance $\bar{r}$ is replaced by an electrical resistance $R=\bar{r}$. Mechanical elements in series are represented by electrical elements in series; parallel mechanical elements by parallel electrical elements. If there is any question as to the validity of the equivalent circuit, Kirchhoff's laws may be applied to the junctions and meshes of the circuit and these should yield equations which are equivalent to those which will be obtained on applying the force law and the velocity law to the mechanical system. Kirchhoff's first law for the currents flowing to any junction is quite analogous to the force law for the forces acting on any junction; Kirchhoff's second law for the e.m.f.'s around a mesh is analogous to the velocity law for the velocities around a closed mechanical circuit.

Having arrived at this much of an understanding of the mechanical problem we might consider abandoning the analogy and working the mechanical problem directly by an application of the force and velocity laws. In case it is necessary to write down Kirchhoff's equations for the electrical circuit and to solve them, little is gained by drawing the equivalent circuit; the analogous equations can be written down at once for the mechanical system. Many well-known electrical laws as, for instance, the reciprocal theorem and Thevenin's theorem, can be

converted by the analogy into equivalent mechanical laws and used directly. In this manner, the electrical technique for the solution of problems can be used without converting the problem to an electrical circuit. In case the properties of the analogous electrical circuit have already been worked out so that no equations need be written or solved, then there is a decided advantage in working with the electrical analogue.

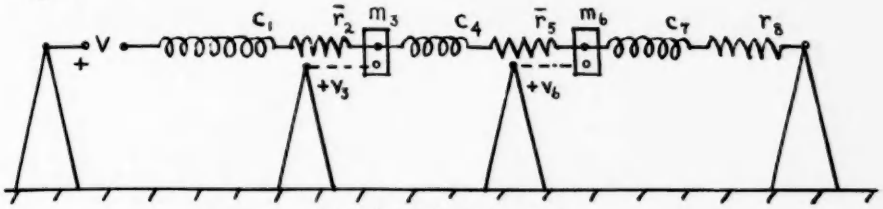


FIG. 10. *Mechanical system.*

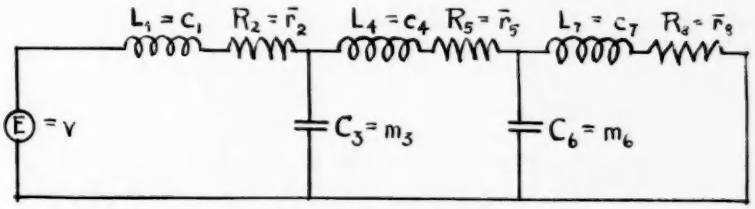


FIG. 11. *New analogue.*

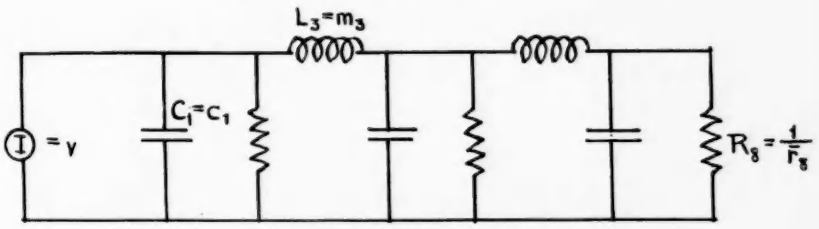
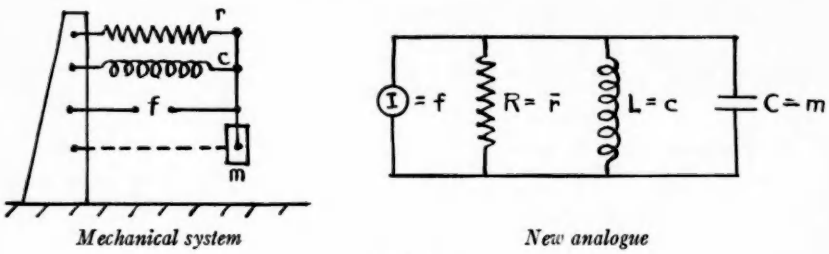


FIG. 12. *Conventional analogue.*

As an example consider the linear mechanical system shown in Fig. 10 in which an impressed velocity is operating and we might wish to know the velocity across (or of) m_6. The new analogue, derived as explained above, is shown in Fig. 11, and the electrical circuit looks much like the mechanical system. The e.m.f. across c_6 will be the desired velocity across m_6. The conventional analogue is shown in Fig. 12 and the circuit is just the opposite of what one would at first glance expect. C_1 and R_2 must be connected in parallel because c_1 and $\bar{r}_2$ have the same force through them. L_3 must be in series with the high wire even though m_3 has one terminal to earth because the force through m_3 is the differ-

ence between the forces through c_4 and c_1. The current through L_6 is the desired velocity across m_6.

Or suppose we wish to solve the well-known problem of the forced vibration of a mass, spring, and resistance in parallel, as shown in Fig. 13, wishing to find the velocity across the mass. The new analogue is a



Mechanical system *New analogue*

FIG. 13.

constant current generator paralleled by a resistance, inductance and capacity, and our analogous problem is to find the e.m.f. across the condenser C.

$$E = IZ = I\,\frac{1}{1/R + 1/i\omega L - \omega C/i} = \frac{I}{1/R + i(\omega C - 1/\omega L)}.$$

Now putting in the mechanical values of the analogous electrical quantities we have

$$v = \frac{f}{1/\bar{r} + i(\omega m - 1/\omega c)} = f\bar{z}$$

which is recognized as being the correct result when we remember that $\bar{r}$ is the reciprocal of the mechanical resistance as usually defined.

THE NEW ANALOGUE OF ELECTROMECHANICAL DEVICES

In treating electromechanical devices, such as microphones and loudspeakers, by converting their mechanical parts to analogous electrical structures which are to be suitably coupled to the electrical system, it must be remembered that in the analogies set forth above, the c.g.s. units were used on the mechanical side while practical units were used on the electrical side. This results in power in ergs per second being analogous to power in watts, thus differing by a factor of 10^7. As long as we deal in analogies only, there is no harm in having the powers in the analogous electrical circuit be 10^7 times the powers in the mechanical system; but when an electrical circuit is coupled to an analogue circuit, the energy leaving the electrical circuit must equal the energy

entering the analogue circuit. One way to resolve this difficulty is to use as our unit of force $(10^7)^{1/2}$ dynes, analogous to one ampere; and as our unit of velocity $(10^7)^{1/2}$ kines, analogous to one volt. The use of these large units will not change the magnitudes of our mechanical impedances, which involve the ratio of force and velocity, but will cause the power, which is the product of force and velocity, to be expressed in watts.

Consider for example a generator connected to a telephone receiver whose diaphragm may be idealized as a mass connected through a spring to the receiver case which, for simplicity, is considered as stationary. There is a two-way interaction between the electrical circuit and this mechanical system: a current through the magnet produces a force on the diaphragm, whose amount in dynes per ampere is assumed known; also any motion of the diaphragm generates an e.m.f. in the magnet circuit, whose amount in volts per kine need not be known. This coupling between the electrical circuit and the analogue circuit can be represented by an ideal transformer whose primary is in series with the electrical circuit and whose secondary is in parallel with the condenser and inductance which represent the effective mass and compliance of the diaphragm. The ratio of primary to secondary turns is the force factor of the magnet system as expressed in large dynes ($(10^7)^{1/2}$ dynes) per ampere. The resulting circuit may be solved in the usual manner to find the voltage across the condenser; this will be the velocity of the diaphragm expressed in large kines ($(10^7)^{1/2}$ kines).

If the electromechanical coupling is electrostatic instead of electromagnetic, then voltage in the electrical circuit results in a force on the mechanical system, that is, it results in a current in the analogue circuit. Also the velocity of the mechanical system generates a current in the electrical circuit. This calls for an inverse transformer to couple the electrical circuit and the analogue circuit of such a nature that the current through the secondary is a constant times the voltage across the primary while the current through the primary is a constant times the voltage across the secondary. This is not a familiar device but it can be replaced by an ideal transformer if either the circuit connected to its primary or to its secondary be replaced by an inverse network with respect to one ohm.[7] Changing a new analogue to its inverse network with respect to one ohm is equivalent to a change to the old analogue. In this problem of electrostatic coupling, the old analogy is advantageous

[7] See Shea, *Transmission Networks and Wave Filters*, p. 311, or Johnson, *Transmission Circuits for Telephonic Communication*, p. 230.

to the extent that it permits an ideal transformer to be used in coupling the electrical circuit to the analogue circuit.

THE ACOUSTICAL-ELECTRICAL ANALOGY

Acoustical transmission systems are not analogous to mechanical systems in quite such a simple manner as one might at first thought suppose. In such an acoustical transmission system as an acoustical filter, each main conducting tube has as its electrical analogue, an electrical line (two wire) having two input and two output terminals.[8] If a tube is used as a side branch with either a closed or an open end, it is as if the input terminals of the equivalent line were connected to the main conducting line while the output terminals were open circuited or short circuited.

The two variables usually used in the discussion of an acoustical transmission system are the sound pressure p at a surface and the volume velocity V through the surface. The acoustic impedance, Za, on a given surface, is defined as the complex quotient of p and V. Thus from the similarity of the following equations the conventional acoustical-electrical analogy may be derived.

$$V = p/Z_a \quad I = E/Z.$$

However, by defining the acoustical bar impedance as V divided by p we could also write

$$V = p\bar{z}_a \quad E = IZ$$

thereby obtaining a new analogy.

Between the two mechanical-electrical analogies previously mentioned, it was possible to make a choice as to the one which was most complete. This was possible because it is easy to tell when mechanical elements are in series and when in parallel; also a strict analogue to the Kirchhoff relations was found with the one analogy only. But if a number of acoustical transmission tubes terminate or originate in a common junction point, there is no *a priori* criterion as to whether they are connected in series or in parallel within the junction. The case is similar to a number of electric lines entering a common junction box; they may be connected either in series or in parallel within the box. If one first assumes the conventional analogy, then he will say that the tubes are in parallel within the junction because they are subject to a common sound pressure; but if he assumes the new analogy, he will

[8] W. P. Mason, B.S.T.J. **6**, 258 (1927).

say that they are in series, for the same reason. There is some justification for a preference of the conventional acoustical-electrical analogy in that the sum of the volume displacements to any junction is zero analogous to Kirchhoff's second law for the e.m.f.'s around a mesh. On the other hand, the new analogy seems more rational in that with it, a tube having an open end is represented by an open circuited line, while a tube with a closed end is represented by a short circuited line; in the old analogy these relations are reversed.

Thus for acoustical systems, the two analogies seem about equally good.

CONCLUSION

The conventional mechanical-electrical analogy is incomplete in the following respects:

I. There is a lack of analogy in the use of the words "through" and "across" which indicates a fundamental difference in the nature of the analogous quantities, for instance, force through and e.m.f. across.

II. Mechanical elements in series must be represented by electrical elements in parallel, and *vice versa*.

III. Mechanical impedances in series must be combined as the reciprocal of the sum of the reciprocals while electrical impedances in series are additive.

IV. There is an incompleteness in the mechanical analogues of Kirchhoff's laws.

The new analogy is free from the above difficulties, which makes it much easier to apply and understand. By considering each mass as having one terminal fixed to the frame of reference, every mechanical system is reduced to a multiplicity of closed mechanical circuits thereby preparing the way for the application of the electrical analogy. In the new analogy, the equivalent electrical circuit can be drawn with ease.

Even those who have never used any analogy and never intend to do so, will be adversely affected by the fact that the old analogy has been used in making the definition of mechanical impedance. This unfortunate choice of definition will retard the development of vibration theory and make it all the more necessary to reply on electrical circuit theory by analogy.

The author nominates for oblivion the conventional left-handed mechanical-electrical analogy.

I wish to thank Mr. L. D. Montgomery and Mr. Earl Burns for their valuable suggestions.

September 12, 1932

BOOK REVIEWS

Acoustics and Architecture. PAUL E. SABINE (Director of Acoustical Research, Riverbank Laboratories, Geneva, Illinois). Cloth, 6×9, 327 pages, 140 drawings and photographs. McGraw-Hill, 1932, $3.50.

The preface states that the book is intended for engineers, technicians and architects. The first three chapters give a general discussion of the nature of sound, the propagation of sound waves, and sustained waves in enclosed spaces. Chapters IV–V deal with reverberation, both theoretical and experimental; while Chapters VI–VII discuss the measurement of sound absorption coefficients and the results obtained. Reverberation in rooms and auditorium design are dealt with in Chapters VIII–IX. The remaining Chapters X–XIII discuss noise, sound transmission and machine isolation. The appendix includes tables of sound absorption coefficients, and sources and loudness of noise.

When reading the book, one is impressed by Dr. Sabine's grasp of the subject, as evidenced by his logical presentation of the mathematical theory and his critical references to the researches of other invesigators. Starting with the equation of simple harmonic motion, Dr. Sabine outlines the development of the expressions for the displacements, velocities and accelerations of the particles in a wave, with formulas for the corresponding pressures and energies. Especially to be commended is his approach to the rather puzzling reverberation equation for rooms by explaining first the easily visualized case of one-dimensional waves in a tube. The development of these equations makes the reader realize anew that any clear understanding of acoustical phenomena, with numerical solutions of problems, requires the use of mathematics.

Another feature of the book is the exposition of the author's own work. In this connection, Dr. Sabine states frankly in the preface: "If undue prominence seems to be given to the results of work done in this country and particularly to that of the Riverbank Laboratories, the author can only plead that this is the work about which he knows most." He therefore gives discussions of the methods of measurement of sound absorption and sound transmission used at Riverbank, together with extensive tables of results obtained; but includes also surveys of the results obtained by others. Concerning sound absorption, he early showed that small areas of samples gave too large coefficients because of the exposed edge effects. He also called attention to the effect of humidity on absorption. In sound transmission, he proved by many data that the controlling factor of partitions in stopping sound was the mass. Theoretically, the elasticity of the partition is equally important, but its effect in partitions in buildings is practically negligible because the frequency of sounds in buildings is usually higher than the natural frequency of the partition.

If Dr. Sabine is a bit controversial about disputed questions, it is only natural that he should defend his own point of view. For example, in the measurement of sound absorption coefficients, he is inclined to favor the experimental procedure inaugurated by Professor Sabine, who used organ pipes for sound sources and a large slowly moving reflector to shift the standing wave pattern in the room. But he uses also the more recent electrical sources of sound, both steady and warble (or "flutter," as he calls it) tones, with a comparison of the results obtained. Since there is no general agreement among investigators at the present time regarding the experimental procedure for measuring absorption coefficients, Dr. Sabine's experiments give information that is helpful in the attempts to find the best solution.

The mathematical portions of Dr. Sabine's book will appeal particularly to engineers, but the architect is not neglected. Chapter VIII gives numerical examples for various types of rooms, showing how the reverberation may be controlled, this being the most important factor in adjusting the acoustics. Further discussions give guidance in regard to curved walls, balcony spaces, radio studios, sound-recording rooms, etc. A formula is given whereby an approximate value of the absorption of a room may be calculated without the usual tedious estimation

of all the surface areas, namely, $-a=0.3V^{\frac{2}{3}}$; or, in words, the approximate absorption of rooms having the usual interior surfaces of masonry walls and ceiling, wood floors, and having seats of 0.3 units each, is given by 0.3 of the volume of the room to the $\frac{2}{3}$ power. While these rules and data will be of use to architects, for guidance in the simple cases, it is becoming more and more apparent that an acoustical engineer is needed for the solution of the more extended problems, in much the same degree that engineers are needed for the adjustment of the lighting, heating and ventilation; except that the acoustics includes an artistic element that adds to the aesthetic value of the room.

In addition to the discussions of his own work, Dr. Sabine takes occasion in many instances to set forth the results obtained by his cousin, Professor Wallace Sabine. In Chapter V is given a connected account of the brilliant series of experiments by which the latter established the fundamental laws of reverberation upon which most of the subsequent work in this field by others has been based. Professor Sabine's original account of this work appeared in a series of articles in architectural journals. These were republished after his death in the volume of *Collected Papers on Acoustics* issued by the Harvard University Press under the editorship of Theodore Lyman. The summarized account given in Chapter V should prove helpful to the student who is interested in a more than theoretical knowledge of the subject, and is at the same time a tribute to the man whose outstanding contributions all workers in the field of applied acoustics gladly recognize.

Altogether, the book is a satisfactory contribution to the subject of architectural acoustics. Engineers will profit by the logical presentation of the mathematical developments together with the many tables of data, and architects will find guidance for the acoustical design of auditoriums and the control of reverberation.

F. R. WATSON

Vibration Prevention in Engineering. ARTHUR L. KIMBALL (Research Laboratory of the General Electric Company), John Wiley and Sons, $2.50.

This book is principally concerned with a quantitative treatment of problems rising from the vibration of rotating machinery. With the view to making it practical the author has incorporated, to a considerable extent, the theoretical and conceptual methods which have been so fruitful in acoustical and communication engineering. The attack is quite directly on a number of practical problems. The general utility to the practicing engineer lies to a great extent in the avoidance of complex theoretical rigor.

For the acoustical engineer this book may be thought of as a treatment of problems of subaudible vibrations. Some types of reaction, particularly those of rotation, as they are discussed have not yet been of importance in acoustics, but it is not unlikely that they will sometime be so in one form or another. An introductory discussion is given of the effect of internal friction in solids. This property of materials will no doubt soon assume considerable technical importance not only in the design of machinery, but in acoustical structures and in the general development and study of alloys and other solids.

R. L. WEGEL

Fortschritte der physikalischen und technischen Akustik. DR. FERDINAND TRENDELENBURG. (Berlin-Siemenstadt) 132 pages, 80 illustrations. Akademische Verlagsgesellschaft, Leipsic, 1932. Cost, 4.8M (approximately $1).

This book gives condensed descriptions, with diagrams, of the researches since 1928 which, directly or indirectly, are of importance in applied acoustics. This publication may be considered as a continuation and extension of two earlier contributions by Dr. Trendelenburg, one[1] appearing in 1926 that was limited to the analysis of sound by exact physical methods;

[1] Trendelenburg, Zeits. f. Hochfrequ. **28,** 54, 84 (1926).

and a second one[2] in 1928 that included a larger field of modern acoustics, particularly the electro-acoustics investigations.

After a brief introduction in Chapter I the book gives a discussion in Chapter II concerning the advances in methods of measurement of the analysis of sound; compensation methods for exact measurement of periodic pressure variations according to amplitude and phase; various procedures for the determination of sound field intensities, energy measurements, and measurement of acoustics of rooms. Chapter III gives discussions of receivers and sources of sound; also, musical instruments and their qualities. Hearing and speaking constitute the subject matter of Chapter IV, while sound reflection, absorption and transmission in buildings are discussed in Chapter V. The book concludes with an account of sound recording and tone films.

On reading the book, one notes a considerable variety of subjects, and thus realizes the activity of acoustic research during the past three years. The discussions are well illustrated by diagrams, with practically no mathematical developments, except an occasional necessary formula. Dr. Trendelenburg has given a well-rounded account of the various investigations, not only in Germany, but in other countries, particularly the United States and England. There are 154 references with the articles. Many of the investigations involve the use of electrical apparatus, particularly radio devices, and indicate the necessity for acoustic investigators to have a working knowledge of these instruments.

Dr. Trendelenburg is to be commended for his efforts in collecting this information, and presenting it in a systematic, condensed form in a book. It gives convenient descriptions of recent acoustic experiments, with results and references, which are useful for investigators in this field.

F. R. WATSON

[2] Trendelenburg, Zeits. f. Hochfrequ. **32,** 27, 94, 131, 173, 202 (1928).